Math 3012-A
Summer 2015
Exam 2
1 July 2015
Time Limit: 70 Minutes

Name: _____

This exam contains 8 pages (including this cover page) and 7 questions. There are 31 points in total. Any expression for a number is acceptable—especially if it is a number from the Twelvefold way. There is no need to find a decimal representation. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	4	
2	4	
3	8	
4	6	
5	3	
6	2	
7	4	
Total:	31	

Formal Symbols Crib Sheet

$\neg$	not	$\wedge$	and	$\vee$	or
$\Rightarrow$	implies	$\lightning$	contradiction	$\in$	element of
$\forall$	for all	$\exists$	there exists	$\Leftrightarrow$	equivalence
$\varnothing$	empty set	$\mathbb{N}$	natural numbers	$\mathbb{Z}$	integers
$\mathbb{Z}_+$	positive integers	$\mathbb{Z}_{\geq 0}$	non-negative integers	$\equiv \pmod{n}$	congruence mod n
$\mathbb{Q}$	rationals	$\mathbb{R}$	reals	$\mathbb{C}$	complex numbers
$\times$	Cartesian product	$\subset$	subset	$\backslash$	set minus
$\cap$	intersection	$\cup$	union	$\mathcal{O}$	big-O asymptotic order
2^A	power set of set A	$|A|$	cardinality of set A	A^B	set of functions $B \to A$

Signatures on the Declaration of Independence:

1. John Hancock
2. Josiah Bartlett
3. William Whipple
4. Matthew Thornton
5. Samuel Adams
6. John Adams
7. Robert Treat Paine
8. Elbridge Gerry
9. Stephen Hopkins
10. William Ellery
11. Roger Sherman
12. Samuel Huntington
13. William Williams
14. Oliver Wolcott
15. William Floyd
16. Philip Livingston
17. Francis Lewis
18. Lewis Morris
19. Richard Stockton
20. John Witherspoon
21. Francis Hopkinson
22. John Hart
23. Abraham Clark
24. Robert Morris
25. Benjamin Rush
26. Benjamin Franklin
27. John Morton
28. George Clymer
29. James Smith
30. George Taylor
31. James Wilson
32. George Ross
33. George Read
34. Caesar Rodney
35. Thomas McKean
36. Samuel Chase
37. William Paca
38. Thomas Stone
39. Charles Carroll of Carrollton
40. George Wythe
41. Richard Henry Lee
42. Thomas Jefferson
43. Benjamin Harrison
44. Thomas Nelson, Jr.
45. Francis Lightfoot Lee
46. Carter Braxton
47. William Hooper
48. Joseph Hewes
49. John Penn
50. Edward Rutledge
51. Thomas Heyward, Jr.
52. Thomas Lynch, Jr.
53. Arthur Middleton
54. Button Gwinnett
55. Lyman Hall
56. George Walton

1. (4 points)   (a) What properties are required for a binary relation R on a set X to be a poset?

    (b) Draw a Hasse diagram for the poset $|$ on the set $X = \{1, 2, 4, 5, 8, 10, 20, 40\}$ where $x|y$ if and only if x divides y without remainder.

2. (4 points) (a) What is a *derangement*?

(b) What is the exact probability that a random permutation of the words of

   "We hold these truths to be self-evident that all men are created equal"

   has every word in a different position than it appears in the preamble to the Declaration of Independence? (Count "self-evident" as 1 word.)

3. (8 points) 56 delegates signed the Declaration of Independence. They are listed for your convenience on the back of page 1.

   (a) The delegates represented exactly 13 distinct states. How many possibilities are there for all 56 delegates to be assigned a state so that exactly 13 are represented?

   (b) In how many ways can the delegates be organized into exactly disjoint 5 sets of debating factions?

   (c) How many ways can we partition the delegates into 5 or fewer sets?

   (d) King George III places bounties on delegates. In how many ways may he do this if he cannot exceed his budget of £5000 (but may spend less) and every bounty is a whole number of pounds?

   (e) (Bonus) A committee of five delegates drafted the Declaration including Robert Livingston, who did not sign. Name them and give the probability of correctly guessing at random from the list of 56 delegates.

4. (6 points) Write in closed form the generating function that counts the following:

   (a) the number of binary strings of length n

   (b) the number of ways to make change for n cents using 2 and 3 cent coins

   (c) the number of strings of $\{1, 2, 3\}$ whose digits sum to n

   (d) (Bonus) the number of integer partitions of n into exactly 6 parts

5. (3 points) How many distinct collections of 13 stars can be created from 4 red, 4 blue, and 6 white stars? Assume stars of the same color are identical.

6. (2 points) Consider the fractal sequence whose first four iterations are shown below. The sequence begins with a single black square. The next step is obtained by replacing every square with a pattern of squares of $\frac{1}{3}$ the sidelength as shown.

et cetera
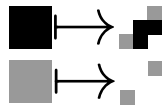
(a) Write a recurrence relation for the total number of squares at stage n.

(b) (Bonus) What is the fractal dimension of the process?

7. (4 points) (a) What is the general solution to the recurrence relation

$$a_{n+1} = a_n + 6a_{n-1}?$$

(b) Find a particular solution to inhomogenous recurrence

$$b_n = 2b_{n-1} - b_{n-2} + 10.$$

The characteristic polynomial is $(A - 1)^2$.