Things I Learned in 3012 Applied Combinatorics
or Why Didn't Shane Just Give Me This Sheet?

I. Sets and Functions

1. A *set* is an unordered collection of other sets, called the *elements* or *members*. For any two sets X, Y either X is an element of Y, $X \in Y$, or not $X \notin Y$. Two sets are equal if they have the same elements.
2. Subset: $X \subset Y$ if every element of X is an element of Y
3. Disjoint: X and Y are disjoint if they have no element in common
4. Set construction (naive):
   i. The empty set $\varnothing$ has no elements
   ii. Metaset: for set X there is a (differnt) set $\{X\}$ with X as an element
   iii. Union: $X \cup Y$ is the set of elements in X or in Y
   iv. Intersection $X \cap Y$ is the set of elements in X and in Y
   v. Ordered pair: (x, y) is the set $\{\{x\}, \{x, y\}\}$
   vi. Cartesian product: $X \times Y$ is the set of ordered pairs (x, y) for all $x \in X$ and $y \in Y$
   vii. Subset specifier: $\{x \in X \mid p(x)\}$ is the subset of X satisfying property p
   viii. Power set: 2^X is the set of all subsets of X
   ix. Natural numbers: Construct non-negative integers $\mathbb{Z}_{\geq 0}$ inductively by defining the next integer n as the set containing all the previously defined integers $0 = \varnothing$, $1 = \{0\}, 2 = \{0, 1\}, \ldots, n = \{0, \ldots, n-1\}$
   x. Constructing numbers leads to the following hierachy of number sets:

   $$\varnothing \subset \mathbb{Z}_{\geq 0} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O}$$

   of natural numbers, integers, rationals, reals, complex numbers, quaternions, octonians
5. Relations: A binary relation on set X is $R \subset X \times X$. We write xRy for $(x, y) \in R$. Common properties of important relations:
   i. Reflexive: $\forall x \in X : xRx$
   ii. Transitive: $\forall x, y, z \in X : (xRy$ and $yRz)$ implies xRz
   iii. Symmetric: $\forall x \in X : xRy$ implies yRx
   iv. Antisymmetric: $\forall x \in X : xRy$ and yRx implies $x = y$
   v. An *equivalence relation* $\cong$ is a reflexive, transitive, symmetric relation on X. Every equivalence relation on X gives a *partition* of X into disjoint subsets whose union is X. The disjoint subsets are called the *parts* of the partition, or the *equivalence classes* $[x] = \{y \in X \mid y \cong x\}$. Every partition gives an equivalence relation, so partitions and equivalence relations of a set biject.
   vi. A *poset* (X, R) is a set X with a *partial order* R which is a reflexive, transitive, antisymmetric relation on X
6. Functions: A function $f : X \to Y$ is a set of order pairs $f \subset X \times Y$ such that for every $x \in X$ there is a unique $y \in Y$ such that $(x, y) \in f$. We write $f(x) = y$ or $x \mapsto y$ for $(x, y) \in f$. Common properties of important functions
   i. *injective* or 1-to-1: $\forall x, x' \in X : f(x) = f(x')$ implies $x = x'$
   ii. *surjective* or onto: $\forall y \in Y :$ there is $x \in X$ such that $f(x) = y$
   iii. *bijective* if both injective and surjective
7. Constructing functions:
   i. identity: every set has an identity 'do nothing' function $id_X : X \to X$ with $id_X(x) = x$ for all $x \in X$
   ii. composition, '"plug in': If $f : X \to Y$ and $g : Y \to Z$ there is a *composed* function $g \circ f : X \to Z$ defined by $g \circ f(x) = g(f(x))$
   iii. inverse: $f : X \to Y$ is a bijection if and only if there is an *inverse* function $f^{-1} : Y \to X$ such that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
   iv. The Axiom of Choice: For any set X of nonempty sets there is a *choice* function $f : X \to \bigcup_{x \in X} x$ such that $f(x) \in x$
8. Cardinality: sets X and Y have the same *cardinality* or number of elements if there is a bijection $f : X \to Y$. Write $|X| = |Y|$, or $|X| = n$ if the integer n bijects to X. Cardinality gives an equivalence relation on any set of sets. The equivalence classes are the *numbers*.

9. Permutations: a bijection $\sigma : X \to X$ is called a permutation of X. The set of all permutations is written $X!$, since $|X!| = |X|!$, or called the *symmetric group* $\mathrm{Sym}(X)$. A *cyclic* permutation is written in cycle notation $(x_0, x_1, \ldots, x_k)$ meaning the function $x_i \mapsto x_{i+1}$ for all $i \in k$ and $x_k \mapsto x_0$ and $x \mapsto x$ for all other $x \in X$. Every permutation may be written as a composition of disjoint cycles.
10. Strings: a length n string on alphabet X is a function $s : n \to X$ which we represent by writing the values of s in order $s_0 s_1 \ldots s_{n-1}$
11. Sequences: usually sequence means an infinite string of numbers $a : \mathbb{Z}_{\geq 0} \to \mathbb{R}$ written $n \mapsto a_n$. But sometimes it just means string.

II. **Counting**

The Twelvefold Way:

$|\{f : k \to n\}|$

How many ways to sort k balls into n boxes?

	Arbitrary any sorting	Injective max 1 ball per box	Surjective each box gets ball
Distinct Balls Distinct Boxes	n^k	$\frac{n!}{(n-k)!}$	$n!\left\{ {k \atop n} \right\}$
Identical Balls Distinct Boxes	$\binom{n+k-1}{k}$	$\binom{n}{k}$	$\binom{k-1}{n-1}$
Distinct Balls Identical Boxes	$\sum_{j=0}^{n} \left\{ {k \atop j} \right\}$	1 if $k \leq n$	$\left\{ {k \atop n} \right\}$
Identical Balls Identical Boxes	$p_{\leq n}(k)$	1 if $k \leq n$	$p_n(k)$

1. Counting Strategies:
   i. Bijection Principle: Sets in bijection are the same size. Biject new problems to old.
   ii. Addition Principle: XOR Disjoint events add $|A \cup B| = |A| + |B|$ if $A \cap B = \varnothing$
   iii. Subtraction Principle: Overcount and subtract the size of the extra. Inclusion-Exclusion:

$$\left| X - \bigcup_{i \in I} A_i \right| = |X| + \sum_{s \subset I} (-1)^{|s|} \left| \bigcap_{i \in s} A_i \right|$$

   iv. Multiplication Principle: ANDTHEN Independent events multiply $|A \times B| = |A||B|$
   v. Division Principle: Overcount and divide out symmetry. See also: Burnside's Lemma.
2. There are $|X|!$ permutations of a set X.
3. Binomial coefficients: There are $\binom{n}{k}$ size k subsets of a size n set

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

4. The Binomial Theorem: For numbers or variables x, y:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

5. The Multinomial Theorem: For numbers or variables $x_0, \ldots, x_k$ and integer powers n:

$$(x_0 + \ldots + x_k)^n = \sum_{j_0 + \ldots + j_k = n} \frac{n!}{j_0! \cdots j_k!} x_0^{j_0} \cdots x_k^{j_k}$$

6. There are 2^n subsets of a size n set.

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

7. Pascal's Relation: $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ allows recursive computation of Pascal's Triangle.

8. A *derangement* of a set X is a permutation $X \to X$ such that for all $x \in X$: $f(x) \neq x$. There are D_n derangements of an n sized set

$$D_n = !n = \text{round}\left(\frac{n!}{e}\right)$$

9. Stirling numbers of the 2nd kind: There are $\left\{ {k \atop n} \right\}$ partitions of a size k set into n parts.

$$\left\{ {k \atop n} \right\} = \frac{1}{n!} \sum_{j=0}^{n} (-1)^j \binom{n}{j} (n-j)^k = n \left\{ {k-1 \atop n} \right\} + \left\{ {k-1 \atop n-1} \right\}$$

10. Bell numbers: There are B_n partitions bijecting to B_n equivalence relations of a size n set

$$B_n = \sum_{j=1}^{n} \left\{ {n \atop j} \right\}$$

11. Generating Functions:
    1. The generating function f of a sequence a as the formal power series

    $$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

    2. Geometric series:

    $$\sum_{k=0}^{\infty} y^k = \frac{1}{1-y}$$

    3. The $\times$ and $+$ Principles apply to generating functions.
    If f (respectively g) is the generating function for the number of outcomes of game A (respectively B) resulting in n points, then $f+g$ is the generating function for ways to play either A xor B and get n points; and fg is the generating function for the ways to play A *and then* B and get n points total
    4. Differentiating term by term provides many series from the geometric:

    $$x \frac{d}{dx} f(x) = \sum_{n=0}^{\infty} n a_n x^n$$

12. Integer partitions:
    i. An integer partition of $k \in \mathbb{Z}_+$ is a non-increasing string of positive integers summing to n. There are $p(k)$ integer partitions of an integer k. These are computable by the generating function

    $$\sum_{n \geq 0} p(k) x^k = \prod_{j=1}^{\infty} \frac{1}{1-x^j}$$

    ii. There $p_{\leq n}(k)$ integer partitions of k into at most n parts (i.e. non-increasing length n or less strings of positive integers summing to k)

    $$\sum_{k=0}^{\infty} p_{\leq n}(k) x^k = \prod_{j=1}^{n} \frac{1}{1-x^j}$$

    iii. $p_n(k)$ integer partitions of k into exactly n parts (i.e. non-increasing length n strings of positive integers summing to k)

    $$\sum_{k=0}^{\infty} p_n(k) x^k = \prod_{j=1}^{n} \frac{x}{1-x^j}$$

    $$p_{\leq n}(k) = p_n(k+n)$$

    iv. The number of integer partitions of k into odd parts and the number of integer partitions into distinct parts are equal

    $$\prod_{j=0}^{\infty} \frac{1}{1-x^{(2j+1)}} = \prod_{k=1}^{\infty} (1+x^k)$$

III. **Induction**

1. Well Ordering Principle: Every nonempty subset of natural numbers has a minimal element.
2. Principle of Mathematical Induction: Let $P : \mathbb{Z}_{\geq 0} \to \{\text{True, False}\}$ be a propositional function. If $P(0)$ is True and for all n we have $P(n)$ implies $P(n+1)$, then $P(n)$ is True for all n.
3. A recursive sequence $a : \mathbb{Z}_{\geq 0} \to \mathbb{R}$ satisfies a *recurrence relation* with a_n given by a formula in terms of earlier terms of the sequence.
4. Factorial:
$$n! = n \cdot (n-1)!$$
5. Greatest common divisor: for $a < b \in \mathbb{Z}_+$
$$\gcd(a, b) = \gcd(a, b - a)$$
6. Many recurrence relations can be solved by manipulating generating functions
7. A k^{th} order linear recurrence relation writes a_{n+k} as a linear combination of the previous k terms
$$a_n = \sum_{j \in k} c_j a_{n+j}.$$
   To compute the solution to a k^{th} order linear recurrence relation:
   1. Plug in $a_n = x^n$ to find and factor the characteristic polynomial is
   $$0 = x^k - \sum_{j \in k} c_j x^j = \prod_{j \in k} (x - \lambda_j)$$
   2. If eigenvalue λ is repeated m times as a root of the characteristic polynomial, the sequences $a_n = \lambda^n$ and $a_n = n\lambda^n$ and ... and $a_n = n^{m-1}\lambda^n$ are solutions to the recurrence.
   3. All possible solutions to the recurrence relation are given by linear combinations of these k different sequences.
   4. Solve for the coefficients of the linear combination using any k known terms of the sequence.

IV. **Pigeonhole Principle**

1. If there is an injection $A \hookrightarrow B$ then $|A| \leq |B|$
2. If k balls are sorted to n boxes then there is a box which has at least $\lceil \frac{k}{n} \rceil$ balls (ceiling $\lceil \rceil$)

V. **Permutation Groups**

1. A *permutation group* or *symmetry group* or just *group* is a nonempty set G of permutations of some set A such that:
   i. G is closed under composition: if $g, h \in G$ then $h \circ g \in G$
   ii. G has inverses: if $g \in G$ then $g^{-1} \in G$
2. A group can *act* on a set of functions $X \subset \{A \to B\}$ by precomposition or a set of functions $Y \subset \{B \to A\}$ by postcomposition. More generally, a group action on a set X is a function $\phi : G \to X!$ such that $\phi(id_G) = id_X$ and $\phi(h \circ g) = \phi(h) \circ \phi(g)$.
3. If f, f' are functions $A \to B$ and $f = f' \circ g$ for some $g \in G$ then we say that they are equivalent modulo G or $f \cong f' \mod G$
4. Orbit or equivalence class of f is all the functions equivalent to f'
$$[f] = \text{orb}_G(f) = \{f' \in X \mid \exists g \in G : f' \circ g = f\}$$
5. The set of equivalence classes or the quotient is the set of distinct things after identifying by the symmetry
$$X \bmod G = X/G = \{[f] \mid f \in X\}$$
6. Stabalizer of f is all the permutations of G whose action does not change f
$$\text{stab}_G(f) = \{g \in G : f \circ g = f\}$$
7. Fix set of g is all the functions which g leaves unchanged
$$\text{fix}(g) = \{f \in X : f \circ g = f\}$$

8. Burnside's Lemma: If G acts on X then

$$|X \bmod G| = \frac{1}{|G|} \sum_{g \in G} |\text{fix}(g)|$$

9. Polya cycle index: if a permutation σ has a_j size j cycles then its cycle index is

$$\prod_j x_j^{a_j}$$

and the cycle index of the group is the average of its elements' indices.

VI. **Complexity**

1. If $f, g : \mathbb{Z}_{\geq 0} \to \mathbb{R}$ are functions we say $f = \mathcal{O}(g)$ if there are constants n_0, c such that for all $n \geq n_0$:

$$|f(n)| \leq c|g(n)|$$

2. $f = o(g)$ if

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

3. $= \mathcal{O}$ is a reflexive transitive relations on any set of functions. Considered as classes of functions

$$\mathcal{O}\left(\frac{1}{n}\right) \subset \mathcal{O}(1) \subset \mathcal{O}(\log n) \subset \mathcal{O}(n^k) \subset \mathcal{O}\left(2^{n^k}\right)$$

4. Given a computing model and a set of basic operations, the *computational complexity* of an algorithm is the maximum number of operations required to perform the algorithm, as a function of the size of the algorithm's input. Typically only the complexity asymptotic class $\mathcal{O}$ is considered to compare between computing models.
5. A *descision problem* is a problem that can be phrased as a TRUE or FALSE question of the inputs.
6. **P** or POLYNOMIAL TIME is the class of all decision problems for which there exists an algorithm with complexity $\mathcal{O}(n^k)$ for inputs size n and some constant k.
7. **EXP** or EXPONENTIAL TIME is the class of all decision problems for which there exists an algorithm with complexity $\mathcal{O}(2^{n^k})$ for inputs size n and some constant k.
8. A *certificate* or *witness* is a proposed solution to a decision problem.
9. **NP** or NONDETERMINISTIC POLYNOMIAL TIME is the class of all decision problems for which there exists an algorithm to *certify* or check certificates to the problem with complexity $\mathcal{O}(n^k)$ for a size n certificate and some constant k.
10. **NP-complete** A decision problem D is **NP-complete** if every problem in **NP** can be reduced to the problem D by an algorithm of complexity $\mathcal{O}(n^k)$
11. $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$. Certainly $\mathbf{P} \subset \mathbf{NP} \subset \mathbf{EXP}$, but perhaps the most important unsolved problem in mathematics is if you can check a problem's solution quickly, can you also solve the problem quickly?

VII. **Posets**

1. A poset (X, R) or partially ordered set is a set X together with a relation R which is reflexive, transitive, and antisymmetric (see relation)
2. A subposet (X', R') of (X, R) a poset such that $X' \subset X$ and $R' \subset R$
3. x covers y if xRy and there is no z such that $xRzRy$
4. x and y are comparable if xRy or yRx. Otherwise they are incomparable.
5. The Hasse Diagram or the cover graph is the digraph with vertex set X and arcs xy whenever x covers y
6. A chain is a subset of X where all elements are mututally incomparable.
7. An antichain is a subset of X where all no two elements are comparable.
8. The size of the largest chain is called the poset height.
9. The size of the largest antichain is called the poset width.

10. Dilworth's Theorem: For a poset (X, R) of height h and width w, X can be partitioned into h antichains, but no fewer, or w chains, but no fewer.
11. A chain partition of poset (X, R) with w chains can be computed by a network flow. Make a network N with two copies X_0 and X_1 of X as vertices and a capacity 1 arc xy from $x \in X_0$ to $y \in X_1$ if xRy and $x \neq y$. Add a source vertex s with a capactity 1 arc to every vertex of X_0, and add a sink vertex t with a capactity 1 arc to every vertex of X_1. Compute a maximum flow ϕ on N. Partition X with x and y in the same chain if $\phi(xy) = 1$.

VIII. **Graph Theory**

1. A *graph* $G = (V, E)$ is a set of vertices V with a set of edges $E \subset \{e \subset V \mid |e| = 2\}$ of 2 element subsets of V.
2. A *subgraph* of G is a graph $G' = (V', E')$ such that $V' \subset V$ and $E' \subset E$.
3. A *complete* graph has all possible edges. K_n has n vertices and $\binom{n}{2}$ edges.
4. If $G = (V, E)$ and $G' = (V', E')$ are graphs then a *graph isomorphism* is a function $f : V \to V'$ such that $\{v_1, v_2\} \in E \Leftrightarrow \{f(v_1), f(v_2)\} \in E'$. If an isomorphism exists then we say $G \cong G'$ are isomorphic. "Isomorphic" is an equivalence relation on sets graphs. Deciding if two graphs are isomorphic is **NP**
5. The set of isomorphisms from G to itself is a group called the symmetry group or automorphism group of G.
6. A *pseudograph* also can allow multiple edges between the same vertex pair or loops at a single vertex.
7. A *digraph* (V, A) or directed graph has directed edges called arcs $A \subset V \times V$
8. A *weighted graph* is a graph with a function $w : E \to \mathbb{R}_+$ assigning a weight to each edge.
9. Edges are *incident* if they share a vertex. Vertices are *adjacent* if they share an edge.
10. If the vertices are ordered $v_0, \ldots, v_{n-1}$ the adjacency matrix A is the matrix with A_{ij} giving the weight or number of edges between $v_i v_j$
11. The *degree* of a vertex is the number of incident edges. $\deg v = |\{e \in E | v \in e\}|$
12. A *walk* length n in G is a sequence of vertices $w = v_0 v_1 \ldots v_n$ such that $\{v_i, v_{i+1}\} \in E$ is an edge.
13. Walks with additional properties have special names
    i. A *trail* contains only distinct edges
    ii. A *circuit* is a closed trail (begins and ends with same vertices)
    iii. An *Eulerian* trail/circuit contains every edge exactly once
    iv. A *path/open walk* contains only distinct vertices
    v. A *cycle* is a closed path (contains only distinct vertices except the first and last are equal)
    vi. A *Hamiltonian* cycle/path contains every vertex exactly once
14. A graph is *connected* if for any two vertices $v, v' \in V$ there is a walk $w = v_0 \ldots v_n$ with $v_0 = v$ and $v_n = v'$.
15. Eulerian graph is a graph that contains an Eulerian circuit
    i. A pseudograph has an Eulerian circuit if and only if it is connected and every vertex has even degree.
    ii. Fleury's Algorithm: print a Eulerian circuit by: Iteratively tranverse and remove edges whose removal will not disconnect the graph
    iii. Postman Problem: the least weight walk traversing every edge of the graph can be computed by finding the minimal set of paths pairing odd degree vertices, adding copies of those edges, and applying Fleury to the resulting Eulerian multigraph. Computable in **P**.
16. Hamiltonian graph is a graph that contains a Hamiltonian cycle
    i. The Hamiltonian graph problem is **NP-complete**
    ii. Dirac's Theorem: If $|V| \geq 3$ and every vertex has degree at least $\frac{|V|}{2}$ then G is Hamiltonian.
    iii. Traveling Salesman: **NP-complete** decision problem. Does a weighted graph have a spanning walk of weight $\leq c$?
17. Graph G is *bipartite* if the vertices can be divided into two disjoint sets $A \cup B = V$ and $A \cap B = \varnothing$ with every edge from A to B, i.e. $E \subset A \times B$
18. Graph G is a *tree* if it is connected and contains no cycle. G is a tree if and only if it is connected and $|E| = |V| - 1$.
19. A *spanning tree* of graph G is a subgraph of G which is a tree and contains every vertex.
20. Prüfer: Trees on vertex set V biject to strings of V with length $|V| - 2$

21. Kirchoff: If G has adjacency matrix A, then let

$$L = \begin{pmatrix} \deg(v_1) & & \\ & \ddots & \\ & & \deg(v_n) \end{pmatrix} - A$$

the *Laplacian* matrix. The number of spanning trees of G is the value of any cofactor of L. (Delete a column and row and then take the determinant.)

22. Kruskal: The least weight spanning tree can be computed by a greedy algorithm: iteratively take the least weight edge which does not form a cycle. Polynomial.

23. Dijkstra: Graph distance can be computed by: iteratvely bredth-first rebuilding the graph while tracking the least weight path to each vertex so far. Polynomial.

24. A *planar graph* is a graph representable by a diagram in the plane with no two edges crossing.

25. (Euler Characteristic of the Sphere) A planar graph divides the plane into R regions/faces and $|V| - |E| + R = 2$. Each G is connected with at least 3 vertices, each region has at least three edges so $3R \leq 2|E|$ and $|E| \leq 3|V| - 6$. Similarly a connected, planar bipartite graph with at least 3 vertices satisfies $|E| \leq 2|V| - 4$

26. Two graphs G and G' are *homeomorphic* if G is isomorphic to a graph obtained from G' by adding degree 2 vertices which subdivide edges. "Homeomorphic" is an equivalence relation on sets of graphs.

27. Kuratowski: G is planar if and only if it has no subgraph homeomorphic to $K_{3,3}$ or K_5.

28. Planarity deciding is a **P** problem, and many otherwise otherwise difficult decision problems are **P** for the class of planar graphs.

29. A *k-coloring* is a function from the vertices $f : V \to k$ ("the colors") such that if $v, v' \in V$ are adjacent then $f(v) \neq f(v')$. The minimal k for which G admits a k-coloring is called the chromatic number.

30. For $k \geq 3$ deciding if G is k-colorable is **NP-complete**

31. Four Color Theorem: Every planar graph has chromatic number at most 4.

32. Network Flows:
    i. A *flow network* is an oriented graph $G = (V, A, s, t, c)$ with distinguished vertices s source and t sink/terminus, and an integer weight function $c : A \to \mathbb{Z}$ called the capacity
    ii. A flow ϕ on G is a weight $\phi : A \to \mathbb{Z}$ such that
        1) capacity limit $0 \leq \phi(uv) \leq c(uv)$
        2) local conservation if $v \neq s, t : \sum_u \phi(uv) = \sum_u \phi(vu)$
        3) total value conservation $\sum_u \phi(su) = \sum_v \phi(vt)$
    iii. A cut is a partition of $V = S \cup T$ with $s \in S$ and $t \in T$. The capacity of the cut is $c(S,T) = \sum_{u \in S, v \in T} c(uv)$
    iv. Ford Fulkerson, Max-Flow-Min-Cut: The maximum flow value is equal to the minimum cut capacity.
    v. Edmunds-Karp: compute a polynomial time by finding *augmenting* in the potential graph weighted $c - \phi$

33. A *matching* of a graph is a set of edges, none of which are incident to each other. A *perfect* or *spanning* matching has an edge incident to every vertex.

34. Bipartite Matching: A maximal sized matching of a bipartite graph (V, E) can be computed with a flow network N. If the bipartition is $V = S \cup T$ attach a source s with a capacity 1 arc to every vertex in S, orient all edges as arcs from S to T with capacity 1, attach a sink t with a capacity 1 arc from every vertex in S. If ϕ is a maximum flow for N, a maximal matching is $\{\{u, v\} \in E \mid \phi(uv) = 1\}$.

35. Menger's Messengers: The minimum number of edge deletions required to disconnect two vertices v and w in a graph is equal to the maximum number of edge-disjoint paths between them. A maximal sized set of edge-disjoint paths can be computed by a network flow with source v, sink v, and all edges replaced by a pair of capacity 1 arcs in both directions.