Math 3012-A
Summer 2015
Exam 1
10 June 2015
Time Limit: 70 Minutes

Name: ____________________

This exam contains 8 pages (including this cover page) and 8 questions. There are 42 points in total. Any expression for a number is acceptable; there is no need to find a decimal representation. Write explanations or proofs clearly and in complete thoughts. Points are reserved for clarity. Do not include any scratch work that is not part of the proof in the proof. Use the blank side of paper for scratchwork. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	6	
2	2	
3	2	
4	4	
5	10	
6	6	
7	4	
8	8	
Total:	42	

Formal Symbols Crib Sheet

$\neg$	not	$\wedge$	and	$\vee$	or
$\Rightarrow$	implies	$\lightning$	contradiction	$\in$	element of
$\forall$	for all	$\exists$	there exists	$\Leftrightarrow$	equivalence
$\varnothing$	empty set	$\mathbb{N}$	natural numbers	$\mathbb{Z}$	integers
$\mathbb{Z}_+$	positive integers	$\mathbb{Z}_{\geq 0}$	non-negative integers	$\equiv \pmod{n}$	congruence mod n
$\mathbb{Q}$	rationals	$\mathbb{R}$	reals	$\mathbb{C}$	complex numbers
$\times$	Cartesian product	$\subset$	subset	$\setminus$	set minus
$\cap$	intersection	$\cup$	union	$\mathcal{O}$	big-O asymptotic order
2^A	power set of set A	$\lvert A\rvert$	cardinality of set A	A^B	set of functions $B \to A$

1. (6 points) (a) Give the definition of a *bijection*.

(b) Give a bijection between the set of portraits below labeled by 9 and the set

$$N = \{Alito, Breyer, Ginsburg, Kagan, Kennedy, Roberts, Scalia, Sotomayor, Thomas\}$$

Bonus if your bijection is *name*.

(c) How many bijections are there between N and 9?

2. (2 points) Female honey bees have a mother and father, but male bees have only a mother. (E.g. a male bee has 1 parent, 2 grandparents, 3 great-grandparents, ...) Write a recurrence relation for the number of ancestors a bee has n generations back.

3. (2 points) What's wrong with this proof?

> <u>Everyone is the same age.</u>
>
> Proof: Induct on the number of people. Everyone in a set of 1 person is the same age, so the base case of one person holds. Suppose that any k people are the same age. Let $p_1, \ldots, p_k, p_{k+1}$ be $k+1$ people. Then $p_1, \ldots, p_k$ form a set of k people, and so all the same age by the inductive hypothesis. Similarly $p_2, \ldots, p_k, p_{k+1}$ are all the same age. But p_2 is in both sets so all $k+1$ people are the same age. By induction any finite number of people are the same age. $\square$

4. (4 points) Ten points are chosen in an equilateral triangle of side length 1. Prove there must be a pair of points that are no more than distance $\frac{1}{3}$ apart.

5. (10 points) The set of (uncased) English letters is $\{A, B, C, \ldots, Z\}$ and has 26 elements.

   (a) How many length 10 strings of English letters are there?

   (b) How many distinct strings can one formed by rearranging the letters of SASSAFRASTASTE?

   (c) How many strings of English letters never repeat a letter and are in alphabetical order?

   (d) How many length 40 strings of English letters are in alphabetical order and may have repeating letters?

   (e) How many length 40 strings of English letters are in alphabetical order and have every letter appear at least once?

   (f) Bonus: How many people are required to guarantee that there is a set of at least 10 people whose first names begin with the same letter and whose last names begin with the same letter?