1. Suppose that Euler's method is used for the equation $y' = \cos(t) + y^2$. Find an upper bound for the local truncation error if a step size of 0.1 is used on an interval where $|y| < 300$. First you'll need to find a bound for $|y''|$.

2. How many solutions can a first order differential equation have if an initial condition is specified?

3. Determine if the following are exact. If the equation is exact, give the general solution. If the equation is not exact, find an integrating factor to make the equation exact.

$$3xy - y^2 + x(x-y)y' = 0$$
$$2ty - 9t^2 + \left(2y + t^2 + 1\right)\frac{dy}{dt} = 0$$
$$e^x y' = -3e^x y - x$$
$$\frac{2ty}{t^2+1} - 2t + (\ln|t^2+1| - 2)y' = 0$$

4. What type of equation are the following? Choose the correct descriptors for each.

$$y'y = \sin t$$
$$y''' + t^3 y' = y\sin t + 10t$$
$$y'' + y' = y$$
$$\sin(y') = \cos t + t^2$$
$$y^{(5)} - 4y'' + 6y = 0$$

   (a) Equation has order _____

   (b) LINEAR NONLINEAR

   (c) AUTONOMOUS NONAUTONOMOUS

   (d) HOMOGENOUS INHOMOGENOUS with inhomogeneity _____

5. For the following differential equations:

$$y' = y^3(y-6)$$
$$y' = y^4 - 4y^3 - y^2 + 16y - 12$$
$$y' = ((y-1)(y-2))^4$$

   (a) Sketch the phase portrait in the y-y' plane for the equation.

   (b) Sketch a slope field and some solutions to the equation.

   (c) What are the equilibrium points for y? What is the stability of each point?

   (d) Describe the long term behavior of solutions in terms of the initial condition $y(0) = y_0$.

6. A 100 gallon tank starts with 50 gallons of water and 10 lbs of salt. 5 gallons per minute of saltwater at a concentration of 2 lb/gal. When the tank is full, a pressure valve allows 1 gallons per minute of mixed fluid out, and the incoming flow slows to 1 gallon per minute. Write differential equations for the system both before and after the tank is full.

7. A pay day loan can have an anual interest rate as high as 1900% continuously compounded. Suppose you can make monthly payments of \$220. Set up a differential equation and initial condition. How much would you owe in 5 months if you borrowed \$1,000?

8. Use Euler's method with step size $h = 1$ to compute $y(3)$ if $y(1) = -1$ and $y' = 3 - t - y$. Use Euler's improved method to obtain a better estimation of $y(3)$.

9. For what points (t_0, y_0) is the initial value problem $y(t_0) = y_0$ and guaranteed to have a unique solution?

$$(t^2 - 9)y' + 2y = \log|16 - 4t|$$
$$y' = y^{\frac{1}{3}}$$
$$y' = y^2$$

10. Solve the initial value problem with $y(1) = 1$ for the following differential equations

$$y' = 5y + t^3$$
$$y' = t^3 e^y$$
$$t^2 y' = y + 1$$