Worksheet 9, Math 1551, Fall 2017

Sections from Thomas 13^{th} Edition: 4.1,4.2

A Few Definitions and Theorems from Sections 4.1, 4.2

- **Local Extrema:** A function has a **local maximum** at $x = c$ if $f(x) \leq f(c)$ for all x in an open interval containing c. A function has a **local minimum** at $x = c$ if $f(x) \geq f(c)$ for all x in an open interval containing c.
- **Critical Points:** An interior point of the domain of $f(x)$ where $f' = 0$, or where f' is undefined, is a **critical point**.
- **MVT:** If $f(x)$ is a continuous function defined on $[a, b]$, and is differentiable over (a, b). Then there is at at least one point, $c \in (a, b)$, where

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Exercises

1. If possible, sketch a curve or give a formula for a function that has the following properties. If it is not possible to do so, state why. Assume in each case that $f(x)$ is continuous, differentiable, and defined for all values of x.

   (a) $f(x)$ has a local maximum at $x = 0$, and $f'(x) < 0$ over the interval $(-1, 1)$.

   (b) $f(x)$ has a local maxima at $x = 0$ and $x = 1$, $f(x)$ has no local minima.

   (c) $f(x)$ is odd, and has local maxima at $x = 1$ and $x = 2$.

2. Which of the following functions satisfy the conditions of the Mean Value Theorem on the interval $[0, 1]$? For those that do not, state why. For those that do, identify all values of c so that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

   (a) $f(x) = \sqrt{x(1 - x)}$

   (b) $f(x) = |x - 0.5|$

3. For each function below, (a) find all critical points, and (b) find all absolute extreme values and/or endpoint extrema as appropriate.

   (a) $f(x) = \sqrt{3 + 2x - x^2}$

   (b) $h(x) = xe^{-x}$

   (c) $g(x) = \frac{x^2-4}{x^2-16}, \quad \text{on } [-1, 1]$

   (d) $h(x) = \sin^{-1}(e^x)$

   (e) $f(x) = e^x + e^{-x}$

   (f) $g(x) = \begin{cases} 3 - x, & x < 0, \\ 3 + 9x - 6x^2 + x^3, & x \geq 0 \end{cases}$