Worksheet 4, Math 1551, Fall 2017

Sections from Thomas 13^{th} Edition: 2.1, 2.6

Exercises

1. Indicate whether the statement true or false. If it is true, in one or two sentences, explain why. If false, give a counter example or explain why in one or two sentences.

   (a) If $y(t) \to 1$ as $t \to \infty$, then y has the horizontal asymptote $y = 1$, and $y(t)$ is never equal to 1.

   (b) If $\lim_{t\to 2} t^2\, f(t) = \infty$, then $\lim_{t\to 2} f(t) = \infty$

   (c) $\lim_{t\to\infty} \left(t - \sqrt{t^2+16}\right) = \lim_{t\to\infty} \left(t - \left(\sqrt{t^2} + \sqrt{16}\right)\right)$

   (d) $\lim_{t\to\infty} \left(t - \sqrt{t^2+16}\right) = \infty - \infty = 0$

2. If possible, sketch the graph of a function that satisfies the following criteria. If it is not possible to do so, state why. It isn't necessary to give a formula for the functions.

   (a) $f(x)$ is continuous, odd, $f(2) < -1$, $\lim_{x\to\infty} f(x) = -1$

   (b) $g(x)$ is continuous, even, $\lim_{x\to-\infty} g(x) = -2$, and $\lim_{x\to\infty} g(x) = 2$

3. If possible, evaluate the following limits. If they do not exist, state why.

   (a) $\lim_{x\to 5^-} \left(\dfrac{3x}{2x-10}\right)$

   (b) $\lim_{t\to\infty} \ln\left(1 + \dfrac{1}{t}\right)$

   (c) $\lim_{x\to\infty} \left(\dfrac{2+\sqrt{x}}{2-\sqrt{x}}\right)$

4. Identify all asymptotes (horizontal, vertical, oblique) of the function $f(x) = \dfrac{x^3 - 4x^2 + 3x}{3x^2 - 6x}$

5. The position of an object is given by $y(t) = t^2 + 2t$.

   (a) Give an expression for the average speed of the object over the interval $[1, 1+\Delta t]$, where $\Delta t > 0$.

   (b) Use your expression in part (a) to calculate the average speed of the object over the interval $[1, 2]$.

   (c) Use your expression in part (a) to calculate the instantaneous speed when $t = 1$.