Worksheet 3, Math 1551, Fall 2017

Sections from Thomas 13^{th} Edition: 2.2, 2.4, 2.5

Exercises

1. Find the inverse f^{-1} of $f(x) = \frac{\sqrt{x}}{\sqrt{x}-3}$. Find hte domain of f^{-1}.
2. If possible, give at least one example of a function for each of the following cases.
   (a) A function $f(t)$ such that
   $$\lim_{t\to 1^+} f(t) \text{ does not exist}$$
   $$\lim_{t\to 1^-} f(t) \text{ does exist}$$
   Hint: use a piecewise function.
   (b) A function that is defined everywhere but is not continuous at exactly one point on its domain.
3. Indicate whether the statement true or false. If it is true, in one or two sentences, explain why. If false, give a counter example or explain why in one or two sentences.
   (a) If $f(1) = 2$, then $\lim_{x\to 1} f(x) = 2$.
   (b) $e^{\ln x} = x$ for all $x \in \mathbb{R}$.
4. Does $f(x) = x^3 - 4x + 1 = 0$ somewhere in the interval $x \in [0, 1]$?
5. For what value of a, if any, is $g(x)$ a continuous function? Sketch $g(x)$ for your value of a.
   $$g(x) = \begin{cases} \sqrt{x-1}, & 1 \le x < 10 \\ a - x, & x \ge 10 \end{cases}$$
6. Evaluate the following limits.
   (a) $\lim_{h\to 0} \dfrac{1 - 1/h^2}{1 + 1/h^2}$
   (b) $\lim_{x\to 0^-} \dfrac{1}{x} - \dfrac{1}{|x|}$
   (c) $\lim_{x\to 0} x^2 \sin\left(\dfrac{1}{x^4}\right)$
   (d) $\lim_{x\to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x^2 - 4}$