MATH 1551. Review problems for Midterm Exam 1.

These review problems are intended for you to practice. These problems might or might not be similar to the ones on the actual midterm exam. Studying only these problems will not prepare you for the test. Your best resources to prepare for the test: your book, class notes, homework, quizzes, worksheets.

1. Sketch a graph of the piecewise function below. Label at least three points on your graph, and find the domain and range of f.

$$f(x) = \begin{cases} \sqrt{x-2}, & x > 3, \\ x^2 - 2x, & -3 < x \leq 3, \\ \frac{1}{x+2}, & x \leq -3. \end{cases}$$

2. Evaluate the expression $\sin(\cos^{-1}(2x))$.

3. Given the functions $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{3-x}$, determine a formula for the functions $f+g$, f^{-1}, $\frac{f}{g}$, $f \circ g$, $g \circ f$, $(f \circ g)^{-1}$. For each function, find the domain.

4. Determine if the following statements are True or False. In each case, give a short explanation of your answer, or give a counter example if False.

   (a) The graph of $y^2 = x^2$ represents the graph of a function of x.

   (b) $f(x) = x^4 + 3x^2 - 1$ is an even function.

   (c) The range of $f(x) = 2 + \frac{x^2}{x^2+9}$ is $[2, \infty)$.

   (d) $\sin(x - \pi/2) = -\cos x$

   (e) $(4^{\sqrt{2}})^{\sqrt{2}/2} = 2$

   (f) $\ln\sqrt{13.5} = 3\ln 3 - \ln 2$

   (g) $\lim_{x \to -3} \dfrac{2 - \sqrt{x^2 - 5}}{x + 3}$ does not exist.

   (h) $\frac{x+2}{\cos x}$ has removable discontinuities.

5. Evaluate the expression $\sin^{-1}(\log_9 3)$.

6. A bacteria colony initially contains 1000 organisms and doubles in size after 5 days. Determine the number of bacteria present in the colony after 12 days. Use the exponential growth model. Simplify your expression as far as you can without a calculator.

7. Find the inverse function f^{-1} for $f(x) = \sqrt{\frac{e^x}{e^x - 3}}$, and describe the domain and range of f^{-1}.

8. Find all asymptotes for the function $f(x) = \dfrac{x^2 - 6x + 9}{2x^2 - 5x - 3}$.

9. Evaluate the limits below or give an explanation as to why the limit does not exist.

   (a) $\lim\limits_{x\to 0} \dfrac{x - x\cos x}{\sin^2(5x)}$.

   (b) $\lim\limits_{x\to 0} \dfrac{\frac{1}{3+x} - \frac{1}{3}}{x}$.

   (c) $\lim\limits_{x\to 0} \dfrac{1}{x^2}\sin^2\left(\dfrac{x}{2}\right)$.

   (d) $\lim\limits_{x\to 4} \dfrac{x-4}{|x-4|}$.

   (e) $\lim\limits_{x\to 0} \dfrac{x}{x - |x|}$.

   (f) $\lim\limits_{x\to\infty} \dfrac{\cos\left(\frac{1}{x^2}\right)}{x^4}$.

   (g) $\lim\limits_{x\to 0^-} \dfrac{9}{7x^{1/3}}$.

   (h) $\lim\limits_{x\to\infty} 2x - \sqrt{4x^2 - 5x}$.

10. Find all values of A and B so that the function f below is continuous at $x = 1$, but discontinuous at $x = 2$. Justify your answer.

$$f(x) = \begin{cases} Ax - B, & x \le 1, \\ 3x, & 1 < x < 2, \\ Bx^2 - A, & 2 \le x. \end{cases}$$

11. The function f is given by the equation:

$$f(x) = \begin{cases} \sqrt{4 - x}, & x < 0, \\ |x - 2|, & 0 < x \le 3, \\ x^2 + 4, & x > 3. \end{cases}$$

   (a) Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 3} f(x)$, or explain why the limits do not exist.

   (b) On what interval(s) is the function continuous?

   (c) Label any discontinuity as removable, jump or infinity. If there is a removable discontinuity, how might you redefine the function to make it continuous at that value?

12. Find all asymptotes for the functions below.

   (a) $f(x) = 5 - 1/(x-2)$

   (b) $f(x) = \frac{x^2 - x}{(x+1)^2}$