MATH 1551. Review problems for Midterm Exam 2.

These review problems are intended for you to practice. These problems might or might not be similar to the ones on the actual midterm exam. Studying these problems will not prepare you for the test. Your resources to prepare for the test: your book, class notes, homework, quizzes, worksheets.

1. Use the definition of derivative as a limit to find the derivative of

$$f(x) = \frac{6x}{x+5}$$

   What is f'(1)?

2. Does the derivative of $f(x) = |x - 4|$ exist at $x = 4$? Justify your reasoning using the definition of derivative.

3. Find the derivative y' for

   $y = 1/(\cot x + \csc x)$.

   $y = \dfrac{9x - 8}{x^2 + 6}$

   $y = (x^2 \ln x)^4$

   $y = 6^{\log_7 t}$

   $8y^{7/4} + 7xy - 2x = 0$

   $y = \cot^{-1} \sqrt{8t}$

   $e^{2x} = \sin(x + 5y)$

   $y = \cos\left(\frac{t}{\sqrt{t+8}}\right)$

   $y = \frac{3\cos\sqrt{x}}{1-\sin^3 x}$

   $y = \sec(\tan(3 - 4t))$

   $7xy + y^2 = 2x + y$

   $y = \frac{(\theta+2)^{3/2}}{\theta\cos\theta}$

   $y = \ln \dfrac{1}{x\sqrt{x-2}}$

   $y = (x^2 - 5x + 5)e^{9x/5}$

   $y = 9\ln(5\ln x)$

   $y = \left(\frac{1+12z}{4z}\right)(12 - z)$

4. Find the derivatives of all orders of $y = (5x^2 - 1)(2 - x)x$.

5. Suppose that the differentiable function $y = f(x)$ has an inverse, that the graph of f pases through the point $(3, 4)$, and it has a slope of $1/5$ at that point. Find the value of $[f^{-1}]'$ at $y = 4$.

6. Does the graph of the following curve have a tangent at the origin? Justify your answer.

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

7. Find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

$$f(x) = \sqrt{5x + 6}, \qquad (2, 4).$$

8. Find all points (x, y) on the graph of $y = \frac{x}{x-1}$ with tangent lines perpendicular to the line $y = x - 8$.

9. Verify that the point $(0, \pi)$ is on the curve given by

$$x^2 \cos^2 y - \sin y = 0$$

and find the lines that are tangent and normal to the curve at that point.

10. At time t (in seconds), the position (in meters) of a body moving a long the s-axis is

$$s(t) = t^3 + 6t^2 - 9t$$

What is the body's acceleration each time the velocity is zero? What is the speed each time the acceleration is zero? What is the total distance traveled by the body from $t = 0$ to $t = 2$?

11. A model for the total stopping distance of a moving car in terms of speed is

$$s = 1.1v + 0.05v^2,$$

where s is measured in ft and v in mph. The linear term $1.1v$ models the distance the car travels during the time the driver perceives a need to stop until the brakes are applied, and the quadratic term $0.05v^2$ models the additional braking distance once they are applied. Find s and ds/dv at $v = 35$ and $v = 70$ mph, and interpret the meaning of the derivative.