Math 1551M2 **Name:** _____
Fall 2017
Exam 2
20 October 2015
Time Limit: 50 Minutes

This exam contains 4 pages (including this cover page) and 7 questions. There are 22 points in total. Write explanations clearly and in complete thoughts. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	0	
2	12	
3	0	
4	0	
5	0	
6	5	
7	5	
Total:	22	

Formal Symbols Crib Sheet

$f : A \to B$	function with domain A & codomain B	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
f^{-1}	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x\to a}$	limit as x approaches a	$\mathbb{R}$	real numbers
$\lim_{x\to a^-}$	limit from below	(a, b)	open interval a to b
$\lim_{x\to a^+}$	limit from above	$[a, b]$	closed interval a to b
$\subset$	subset of	$\in$	element of
$\cap$	intersection	$\cup$	union
$\mapsto$	maps to	f'	derivative
$\frac{d}{dx}$	derivative with respect to x		

Derivatives Crib Sheet

For constant $a \in \mathbb{R}$ and arbitrary real functions f and g

Function	Derivative	Function	Derivative
a	0	af	af'
$f+g$	$f'+g'$	fg	$f'g+fg'$
$\frac{f}{g}$	$\frac{f'g-fg'}{g^2}$	$f \circ g$	$(f' \circ g)g'$
f^{-1}	$\frac{1}{f' \circ f^{-1}}$	x^a	ax^{a-1}
a^x	$a^x \ln a$	$\log_a \lvert x \rvert$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$	$\csc x$	$-\csc x \cot x$
$\cos x$	$-\sin x$	$\sec x$	$\sec x \tan x$
$\tan x$	$\sec^2 x$	$\cot x$	$-\csc^2 x$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$\operatorname{arccsc} x$	$\frac{-1}{\lvert x \rvert \sqrt{x^2-1}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	$\operatorname{arcsec} x$	$\frac{1}{\lvert x \rvert \sqrt{x^2-1}}$
$\arctan x$	$\frac{1}{1+x^2}$	$\operatorname{arccot} x$	$\frac{-1}{1+x^2}$
$\sinh x$	$\cosh x$	$\cosh x$	$\sinh x$

1. What is the limit definitoin of the derivative of a real function f. What is the geometric meaning of the derivative for the graph of the function? What does the derivative mean in terms of the change of f?

2. Consider $h(x) = \arctan(5x + 2)$.
   (a) (6 points) Find the third derivative $h'''(x)$.
   (b) (6 points) h is invertible for $x > \frac{1}{\sqrt{15}}$. Compute the value of the inverse derivative $(h^{-1})'$ evaluated at $h(1)$.

3. The data table below contains some values of real functions f and g and their derivatives f' and g' evaluated at different values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	2	2	5
1	2	3	1	-4
2	6	1	0	0
3	5	3	3	-1

   (a) What is the average rate of change of f between 0 and 2?
   (b) What is the instaneous rate of change of g at 1?
   (c) What is the derivative of the sum $f + g$ at $0, 1, 2$, and 3?
   What is the derivative of the product fg at $0, 1, 2$, and 3?
   (d) What is the derivative of the composition $f \circ g$ at 0,1,2, and 3?
   (e) What is the derivative of the inverse function g^{-1} at 0,1,2, and 3?
   Give a point where g^{-1} is not differentiable.

4. Consider the curve in the x-y plane defined by all the points (x, y) that satisfy the equation
$$4xy + 3\sin y = x - 1$$
   Compute the equation of the tangent line to the curve at the point $(0, 1)$.

5. Consider the piecewise defined real function g
$$g(x) = \begin{cases} 4 + 2x & \text{if } x < -3 \\ 2 + 2x & \text{if } -3 \le x \le -1 \\ \frac{4}{1+x^2} & \text{if } -1 < x < 1 \\ |2 - x| + 1 & \text{if } x \ge 1 \end{cases}$$
   What is the domain of g'? Compute a formula for the derivative g' at all points where it is defined.

6. (5 points) Compute the limit. (Hint: This limit is a derivative of some function at a point. Compute the derivative and use it to evaluate the limit.)

$$\lim_{x\to 0} \frac{e^{3(1+x)^2} - e^3}{x} =$$

7. (5 points) Explain how you can derive the formula for $\frac{d}{dx} \arctan x$ as a rational function in x from the chain rule and the fact that $\frac{d}{dx} \tan x = \sec^2 x$.