Math 1551-G
Fall 2015
Practice Exam 1
18 September 2015
Time Limit: 50 Minutes

Name: _____

This exam contains 9 pages (including this cover page) and 7 questions. There are 54 points in total. Write explanations clearly and in complete thoughts. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	7	
2	6	
3	8	
4	11	
5	4	
6	6	
7	12	
Total:	54	

Formal Symbols Crib Sheet

$f : A \to B$	function with domain A & codomain B	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
f^{-1}	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x \to a}$	limit as x approaches a	$\mathbb{R}$	real numbers
$\lim_{x \to a^-}$	limit from below	(a, b)	open interval a to b
$\lim_{x \to a^+}$	limit from above	$[a, b]$	closed interval a to b
$\subset$	subset of	$\in$	element of
$\cap$	intersection	$\cup$	union
$\mapsto$	maps to		

1. Consider the function $f(x) = 5\sin(x/8) + 4$

    (a) (2 points) What is the domain of f?

    (b) (2 points) What is the range of f?

    (c) (3 points) Choose a restricted domain for f on which the function is invertible. What is the domain and range of the restricted inverse?

2. (a) (3 points) Find the limit

$$\lim_{x \to -\infty} \sqrt{\log_2 |1 - x^3| - \log_2 |x^2 + 3x^3 - 42|}$$

(b) (3 points) Sketch the graph of the function $2\arcsin(3x + 1) + 1$.
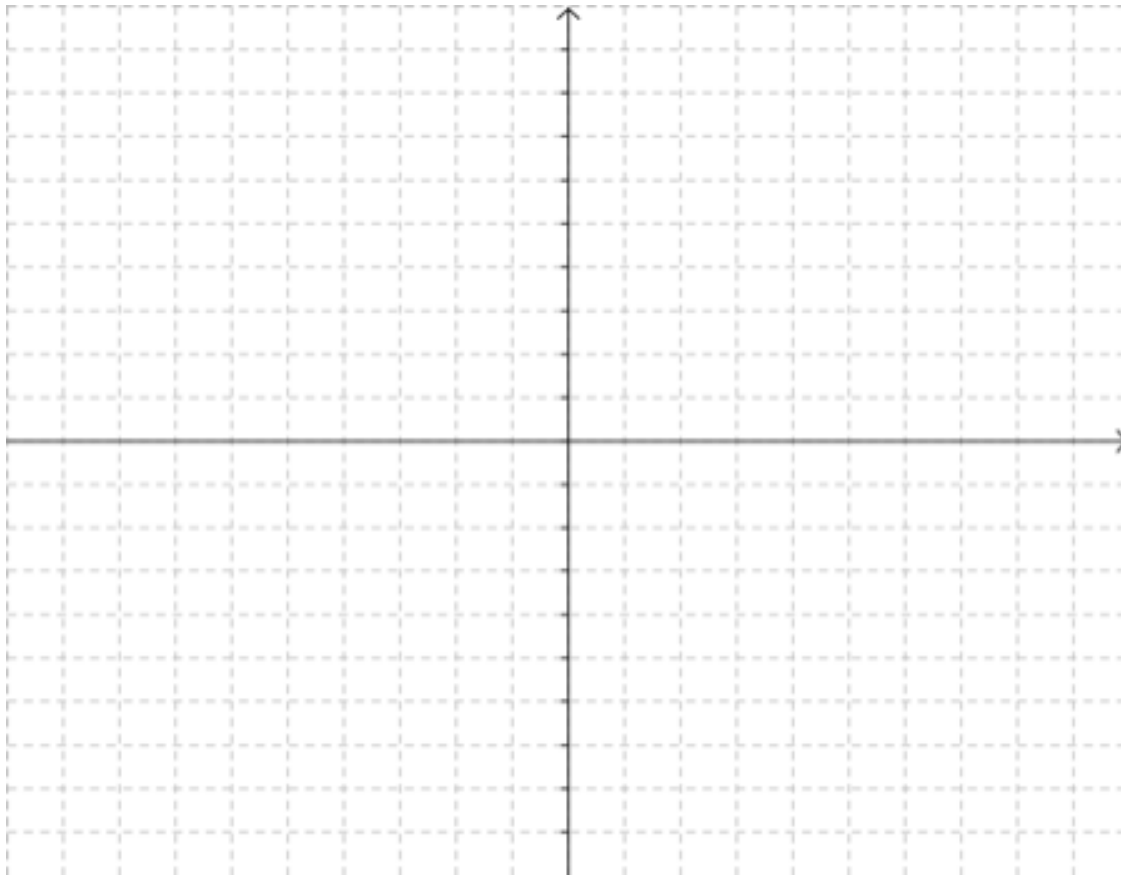
3. Consider the functions $g(t) = \arccos(t^2)$ and $h(t) = \frac{1}{\sqrt{t+4}}$

   (a) (2 points) What is the domain of g?

   Domain of $g =$

   (b) (2 points) What is the range of g?

   Range of $g=$

   (c) (2 points) Is g invertible? Why or why not?

   (d) (2 points) Give the composition function $g \circ h$ in terms of t.

   $g \circ h(t) =$

4. Consider the piecewise defined real function g

$$g(x) = \begin{cases} \sec\left(\frac{1}{x}\right) & \text{if } x < 0 \\ \tan(\pi x) & \text{if } 0 \le x \le 5 \\ \frac{x-4}{\sqrt{x}+2} & \text{if } x > 5 \end{cases}$$

(a) (2 points) What does it mean for a function to be continuous at a real number?

(b) (3 points) Give a real number where the function g is discontinuous. Why is g discontinuous at that number?

$$g(x) = \begin{cases} \sec\left(\frac{1}{x}\right) & \text{if } x < 0 \\ \tan(\pi x) & \text{if } 0 \le x \le 5 \\ \frac{x-100}{\sqrt{x}-10} & \text{if } x > 5 \end{cases}$$

(c) (2 points) Give a real number r where g has a removable discontinuity. What is the value of $\lim_{x \to r} g(x)$?

$r =$

$\lim_{x \to r} g(x) =$

(d) (2 points) Compute the limit

$\lim_{x \to +\infty} g(x) =$

(e) (2 points) Compute the limit

$\lim_{x \to 5^-} g(x) =$

5. (a) (2 points) State the Squeeze Theorem of Limits.

(b) (2 points) Suppose that f is a function such that

$$-\frac{1}{2}x^2 + 4x - 5 \le f(x) \le 3\frac{\sin(x-4)}{x-4}$$

for all $x \in (3, 5)$. Explain what you can conclude about $\lim_{x \to 4} f(x)$.

6. Compute the following limits.

   (a) (3 points)
   $$\lim_{x \to 5} \frac{1 - \cos(x - 5)}{\sin(x - 5)}$$

   (b) (3 points)
   $$\lim_{x \to 0} \frac{\cos(-x) - \cos(x)}{x}$$

7. Find any horizontal, vertical, or oblique asymptotes of the following rational functions.

   (a) (3 points)
   $$\frac{x^4 - 5x^2 + 6}{x^3 - 1}$$

   (b) (3 points)
   $$\frac{2x^5 - 3}{x^2 - 6x + 8}$$

   (c) (3 points)
   $$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)}$$

   (d) (3 points)
   $$\frac{(x-1)(2x-3)}{(2x)(3x+2)}$$