Final Exam Review Exercise Set B: Practice Final Exam, Math 1551, Fall 2017

Instructions

1. Notes, calculators, phones, and books are not allowed.
2. Unless specified otherwise, justify your reasoning for full marks.
3. Enter all requested information on the top of this page.
4. You are required to **show your work** and justify your answers for all questions *except where explicitly stated*.
5. **Organize your work**, in a reasonably neat and coherent way.
6. Calculators, notes, cell phones, and books are not allowed.

Helpful Formulas

$$\sin^2 x + \cos^2 x = 1, \qquad 1 + \tan^2 x = \sec^2 x, \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin^2 x = \frac{1}{2}[1 - \cos(2x)], \qquad \cos^2 x = \frac{1}{2}[1 + \cos(2x)], \qquad \sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin y \sin x$$

$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$$

for grading purposes only

1. (2pts) Circle **true** if the statement is true, otherwise, circle **false**. You do not need to explain your reasoning.

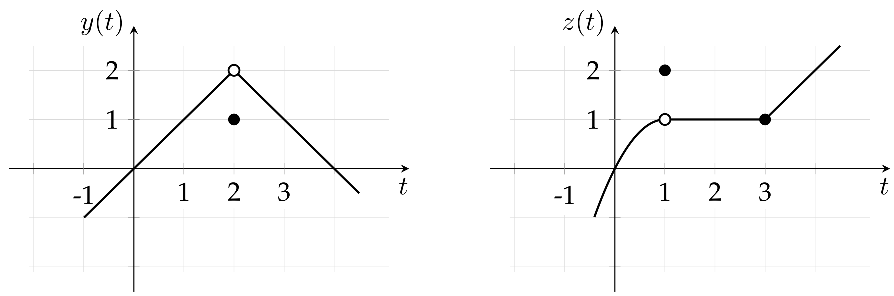
   (a) If $\lim_{x\to 1^+} f(x)$ and $\lim_{x\to 1^-} f(x)$ are both equal to 3, then $\lim_{x\to 1} f(x)$ must exist.

   true **false**

   (b) If $F'(x) = G'(x)$ for all x, then $F(x) - G(x)$ must be a constant.

   true **false**

2. The graph of a function, $y(t)$, is shown on the left. $z(t)$ is shown on the right.

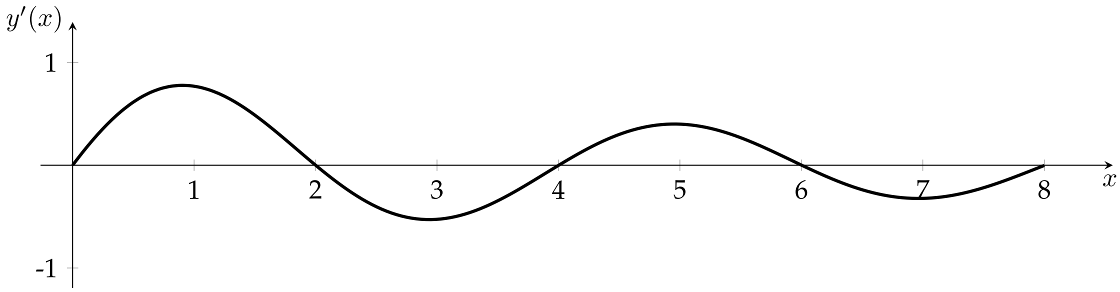


   1 Evaluate: $\lim_{t\to 2} (y(t) + z(t))$

   2 What is the average rate of change of $z(t)$ over the interval $t \in [2, 4]$? Justify your reasoning.

3. (5pts) Identify the absolute maximum value of $y = x \ln x$ on the interval $x \in (0, 1]$, and where the absolute maximum is located.

4. (5pts) The graph below shows the derivative, $y'(x)$, of a function, $y(x)$.



Assume that y is defined for $x \in [0, 8]$.

(a) On what intervals is y increasing?

(b) For what values of x, if any, does y have a local maximum?

(c) On what intervals is y concave up?

(d) For what values of x does y have a critical point?

5. (5pts) If possible, sketch a function $y(x)$, that is odd, concave up on the interval $(0, 2)$, has a local minimum at $x = 1$, and is increasing on $(1, \infty)$. If it isn't possible to do so, explain why. Assume that y is continuous, differentiable, and defined for all values of x. Label your axes.

6. (10pts) Evaluate the limits, if they exist.

(a) $\lim_{x\to 0} \frac{1}{x^2} \cos x$

(b) $\lim_{x\to 4^-} \frac{\sqrt{2x}(x-4)}{|x-4|}$

(c) $\lim_{x\to\infty} (3x - \sqrt{9x^2 - 4x + 9})$

7. (10pts) Construct the equation of the normal line to the curve

$$x^2 + 2xy + y^3 = y^2 + 4x$$

at the point $(0, 1)$.

8. (10pts) Calculate the derivative of the following functions. You do not need to simplify.

(a) $f(x) = \sec^2(\sin^3 x)$

(b) $h(t) = \sqrt{te^{\sin(t/2)}}$

(c) $y = 3t^{1-t^2}$

9. (10pts) One car is approaching an intersection from the north at 40 mi/hr. A second car approaches from the east at 55 mi/hr. Calculate the rate at which the distance between the cars changes when the southbound car is 16 mi away from the intersection and the westbound car is 30 mi from the intersection.

10. (20pts) Consider the function

$$f(x) = \frac{x^3}{x^2 - 1}.$$

(a) Determine the intervals where $f(x)$ is continuous

(b) Determine the points where $f(x) = 0$.

(c) Identify all asymptotes.

(d) Find the critical points.

(e) Write the intervals where f is increasing, decreasing.

(f) Determine the points where the function has extreme values (write coordinate points).

(g) Sketch of the graph of $f(x)$. Label your axes.

11. (10pts) Use the definition of derivative to obtain the derivative of

$$f(x) = \frac{x}{x+1}$$

You may use derivation rules to check your answer, but to get credit, you must obtain the derivative from the definition.

12. (10pts) An open top can is constructed as a right circular cylinder with a volume of 1000 cm^3. Find the dimensions of the can that minimize the surface area.