Section 4.7 : Newton's Method

Chapter 4 : Applications of Derivatives

Math 1551, Differential Calculus

"In mathematics the art of proposing a question must be held of higher value than solving it." – Georg Cantor

In this section we introduce a method for solving a difficult problem. Our algorithm will lead us to more questions than answers.

Section 4.7 Newton's Method

Topics

1. Newton's Method for solving $f(x) = 0$.

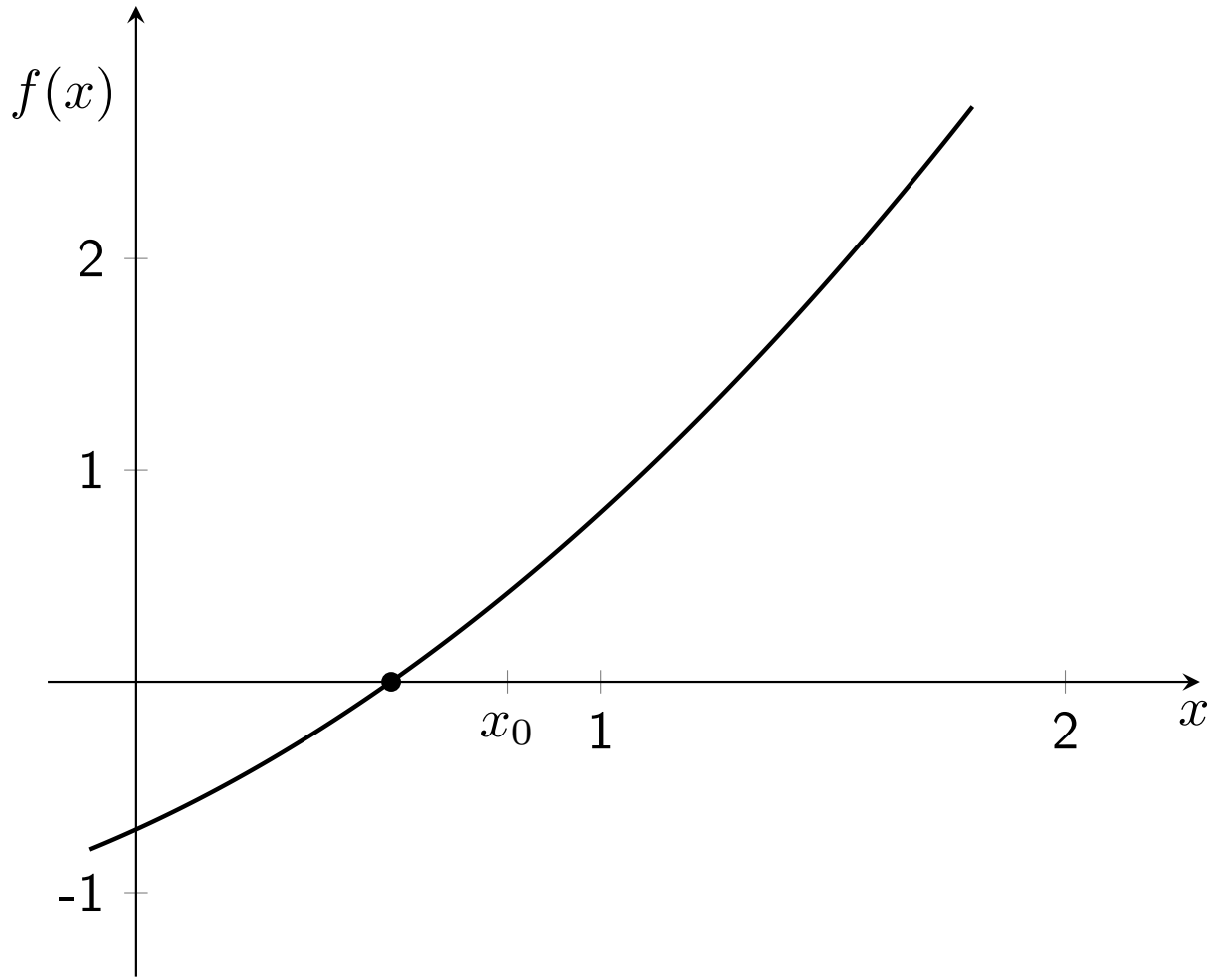
Learning Objectives

For the topics in this section, students are expected to be able to:

1. Given a differentiable function $f(x)$ and an initial estimate x_0, apply one or two steps of Newton's method to estimate a solution to an equation.

Motivation

Suppose we are given a differentiable function $f(x)$ and an estimate of where $f(x) = 0$. We want to identify values of x such that $f(x) = 0$.



Derivation of Newton's Method

Newton's Method

Algorithm

Given x_0 and a differentiable function $f(x)$, a solution to $f(x) = 0$ is estimated with:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

Participation Activity: Index Card

- Please work **by yourself or with one other person**
- Each group submits **one** completed card
- Print full names at the top of your card
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution
- Print today's date at the top, which is _____

The activity consists of one or two of the examples in this lecture. Your instructor will pass out index cards.

Example

Use Newton's Method to estimate a solution to $\sin(x) = x^2 - 1$. Start with $x_0 = 0$ and calculate x_1.

Example

Use Newton's Method to estimate a root of $f(x) = x^3 - 5x + 1$. Start with $x_0 = 1$ and calculate x_1.

Summary

- Given an x_0, Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

- Good questions:
  - Does the method does always converge to a solution?
  - How do we know when to stop the algorithm?
  - When if there are multiple solutions to $f(x) = 0$?
  - What happens if $f'(x_n) = 0$?
  - How would we choose x_0?
- The learning objective for this section: "Given a differentiable function $f(x)$ and an initial estimate x_0, apply one or two steps of Newton's method to estimate a solution to an equation."