Section 4.3 : Monotonic Functions, the First Derivative Test

Chapter 4 : Applications of Derivatives

Math 1551, Differential Calculus

Section 4.3 Monotonic Functions, the First Derivative Test

Topics

- 1. Identifying where functions are increasing and where they are decreasing.
- 2. The first derivative test.

Learning Objectives

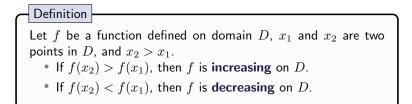
For the topics in this section, students are expected to be able to:

- 1. Determine where a function is increasing or decreasing.
- 2. Classify critical points using the first derivative test.
- 3. Sketch functions using the first derivative and the first derivative test.

Motivation

- In sketching the graph of a function it is useful to know where it increases and where it decreases over an interval.
- This section gives a test to determine where a function increases and where it decreases.
- We also explore one method for testing the critical points of a function to identify whether local extreme values are present.

Increasing and Decreasing Functions



Example

Give a formula for an even function f(x) that is increasing for $x \in (-\infty, 0]$, and decreasing for $x \in [0, \infty)$.

Participation Activity: Index Card

- Please work in groups of two or three
- Each group submits one completed card
- Print full names at the top of your card
- Every student in a group gets the same grade
- Grading scheme per question:
 - $\circ~$ 0 marks for no work or for students working by themselves
 - 1 mark for starting the problem or for a final answer with insufficient justification
 - 2 marks for a complete solution
- Print today's date at the top, which is ______

The activity consists of one or two of the examples in this lecture. Your instructor will pass out index cards.

Example 1

If possible, give a formula for a continuous function, f(x), that satisfies the following criteria. If it is not possible to do so, state why.

- a) Domain D is [0,2], f is increasing on D, f'(x) < 0 on D.
- b) Domain D is [0,2], f is increasing on D, f'(x) = 0 on D.
- c) Domain D is [0,2], f is increasing on D, f'(x) > 0 on D.
- d) Domain D is [0,2], f is decreasing on D, f'(x) < 0 on D.

Derivatives and Increasing and Decreasing Functions

Definitions
Let f be a differentiable function.
If f'(x) > 0 on (a, b), then f is increasing on [a, b].
If f'(x) < 0 on (a, b), then f is decreasing on [a, b].
A function that is increasing (or decreasing) on an interval is monotonic on that interval.

Example 2

Let $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

- a) Determine where the function is increasing, and where it is decreasing.
- $b)\,$ Identify the local extrema of the function and where they are located.
- c) Sketch f(x).

The First Derivative Test

Suppose f has a critical point at x = c.

- If f'(x) changes from positive to negative at c, then f has a **local maximum** at c.
- If f'(x) changes from negative to positive at c, then f has a **local minimum** at c.
- If f'(x) doesn't change sign from positive to negative at c, then f has no local minimum or maximum at c.

Additional Examples (if time permits)

1. If possible, sketch a function f(x) that is odd, continuous, f'(x) < 0 on (-1, 0), local minimum at x = 1.

2.
$$f(x) = x^2 - x - \ln x$$
.

- a) Determine where the function is increasing, and where it is decreasing.
- b) Identify the local extrema of the function and where they are located.
- c) Sketch f(x).