Section 3.10 : Related Rates

Chapter 3 : Differentiation

Math 1551, Differential Calculus

"Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them." - Joseph Fourier

Section 3.10 Related Rates

Topics

1. Related rate problems.

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Solve related rate problems.

Motivation

Recall the learning objectives for our course from the syllabus.

A) **Construct** mathematical expressions and graphs involving functions and their derivatives.

B) **Compute** mathematical quantities using differential calculus and interpret their meaning.

C) **Analyze** mathematical statements and expressions (for example, to assess whether a particular statement is accurate).

D) **Write** logical progressions of precise statements to justify and communicate mathematical reasoning.

E) **Apply** calculus concepts to solve real-world problems such as optimization and related rate problems.

This lecture involves all of the objectives, and focuses on the last item.

Example 1

Two boats, one heading west, the other heading east, are approaching other on separate courses that are 4 miles apart. Both boats are moving at 25 miles per hour. At what rate is the distance between them changing when the boats are 5 miles apart from each other?

Example 2

A conical paper cup, 8 cm across the top and 6 cm deep, is completely full of water. A leak at the bottom appears, losing water at 2 cm^3 per minute. How fast is the water level changing when the water is 3 cm deep?

General Strategy

Solving rate problems tend to involve the following sequence of steps.

1. Read the question.
2. Draw a diagram.
3. Introduce variables.
4. Construct an equation.
5. Calculate derivative at a point.
6. Express answer to question using appropriate units.

Please express final answer with units.

Equation Construction (Step 4)

Related rate problems often rely on equations based on the following.

- Formulas for distance, length, area or volume
- The Pythagorean theorem
- Trigonometric functions
- Similar triangles
- Arc length

The above formulas and relationships will not be given on midterms, quizzes, exams.

In-Class Participation Activity: Worksheet

Some of the examples in these slides are incorporated into a worksheet.

The usual grading scheme applies:

- Please solve worksheet problems in groups of 2 or 3 students
- Each group submits **one** completed worksheet
- Clearly print full names at the top of your sheet
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work, for students working by themselves, or for working in a group of 4 or more
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution

Examples

1. A snowball melts so that its surface area decreases at a rate of 1 cm^2 per minute. Calculate the rate at which the diameter decreases, when the diameter is 10 cm. *Hint: the surface area of a sphere has the formula* $4\pi r^2$.
2. A person 2 m tall walks directly away from a streetlight that is 8 m high, at a rate of 1.5 m/s. How fast is the length of the person's shadow changing as a function of time?
3. A 13-foot ladder rests against a wall, forming an angle θ with the ground. The bottom of the ladder is being pushed towards the wall at a rate of 0.1 feet per second. What is the rate at which θ is changing when the top of the ladder is 12 feet above the ground?
4. An object is moving along the curve $x^2 + 3xy + y^2 = 1$. If $y'(t) = 2$ for all t, compute the possible values of $x'(t)$ when $y = 1$.