Section 3.9 : Inverse Trigonometric Functions

Chapter 3 : Differentiation

Math 1551, Differential Calculus

Section 3.9 Inverse Trigonometric Functions

Topics

1. Derivatives of inverse trigonometric functions.

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Differentiate inverse trigonometric functions.

Students are not expected to memorize the derivatives of inverse trigonometric functions. They'll be given on the cover page of quizzes/midterms/exams.

Example

Example: Differentiate $y = \sin^{-1} x$.

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

Students are not expected to memorize the derivatives of inverse trigonometric functions. They'll be given on the cover page of quizzes/midterms/exams.

Example

Differentiate $y = \left(\tan^{-1}(3x^2)\right)^2$.

Looking Ahead to Integration

- Derivatives of the inverse trigonometric functions are used in integral calculus (Math 1552) in the following way.
- Suppose we need to identify the function whose derivative is

$$\frac{1}{1+x^2}$$

This is an example of an **integration** problem.

Students aren't expected to be familiar with this material. It's presented to motivate derivatives of inverse trigonometric functions.

Additional Examples (as time permits)

Differentiate the following functions.

a) $y = \ln\left(\sec^{-1}(2x)\right)$

b) $y = \sec^{-1}\left(\ln(2x)\right)$