Section 3.7 : Implicit Differentiation

Chapter 3 : Differentiation

Math 1551, Differential Calculus

Section 3.7 Implicit Differentiation

Topics

1. Implicit differentiation
2. Normal lines

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Use implicit differentiation to calculate derivatives.
2. Construct the equation of a normal line to a curve.

Motivation

- So far the functions we've worked with can be expressed **explicitly** in terms of another variable. For example,

$$y = x^3 + 1$$

  The derivative can be found using our derivative rules.
- Suppose we have a relation that defines y **implicitly** as a function of x. For example,

$$y^2 = 1 - x$$

  How would we determine an expression for $y'(x)$?
- We do not yet have a method for differentiation of implicit relations.

Example 1

Construct the equations for the tangent lines to

$$y^2 = 1 - x$$

at $x = -5$. Sketch the curve and the tangent lines.

Implicit Differentiation

Given an implicit relation in x and y, to calculate $y'(x)$:

1. Differentiate both sides of the relation with respect to x, treating y as a function of x.
2. Solve for $y'(x)$.

The Normal Line

Given $y = y(x)$, to construct the equation of the line that is perpendicular to the tangent lane at x_0, we use:

This is the equation of the **normal line**.

Example 2

Construct the equation of the tangent and normal lines to

$$x^3 + y^3 = 6xy$$

at the point $(3, 3)$.