Section 3.6 : The Chain Rule

Chapter 3 : Differentiation

Math 1551, Differential Calculus

Section 3.6 The Chain Rule

Topics

1. The Chain Rule

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Differentiate composite functions using the Chain Rule.

Motivation

If the position of an object were given by

$$s(t) = \sin(t^2 + 1),$$

how would we differentiate this function to determine its velocity and acceleration?

We do not yet have a rule for differentiation of composite functions.

The Chain Rule

Theorem

If $f(u)$ is differentiable at u, and $u = g(x)$ is differentiable at x, then the derivative of the composite function $f \circ g$, with respect to x, is

$$\frac{d}{dx}(f \circ g) = \frac{d}{dx} f(g(x)) = \frac{df}{du}\frac{du}{dx}$$

Proof (if time permits):

Examples

Differentiate the following functions using the Chain Rule.

a) $y(x) = e^{-x^2}$

b) $s(t) = \sin(t^2 + 1)$

c) $h(t) = \frac{\sin^2(3t)}{t^2}$