Section 3.1 : Tangents and the Derivative at a Point

Chapter 3 : Differentiation

Math 1551, Differential Calculus

"A synonym is a word you use when you can't spell the other one."

- Baltasar Gracián

In this section we introduce multiple interpretations of the same concept: the derivative. Like synonyms, sometimes it is more convenient to use one interpretation of the derivative instead of another.

Section 3.1 Tangents and the Derivative at a Point

Topics

1. Review average and instantaneous rate of change.
2. The derivative
3. Computation and interpretation of the derivative.

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Compute and interpret derivatives of functions of a single variable.

Recall: Average Rate of Change

The **average rate of change** of $y = f(x)$ over interval $[x_0, x_0 + h]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0 + h) - f(x)}{h}$$

The line passing through the two points

$$(x_0, f(x_0)) \text{ and } (x_0 + h, f(x_0 + h))$$

is a **secant line**.

Could someone remind us:

1. What does h represent?

2. What is the relationship between $\frac{\Delta y}{\Delta x}$ and the secant line?

Let $h \to 0$

Now suppose we let $h \to 0$.

$$\lim_{h \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

What does this represent?

$f'(x_0)$

Definition: Derivative at a Point

The **derivative** of $f(x)$ at $x = x_0$ is defined as

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Example

The height of a ball is given by $d(t) = 3 - t^2 + 2t$.

a) What is the instantaneous velocity of the ball at $t = 1$ seconds?

b) Plot $d(t)$ to confirm your result in part (a).

Summary

The derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

represents:

- the slope of the tangent line at a point
- the instantaneous rate of change of a function at a point

We use both interpretations throughout this course: sometimes one interpretation is more convenient than another.