# Section 2.6 : Limits Involving Infinity; Asymptotes of Graphs

Chapter 2 : Limits and Continuity

Math 1551, Differential Calculus

*"The only real valuable thing is intuition"* - Albert Einstein

As you are solving problems from this section you will develop intuition for identifying asymptotes. But be sure to justify your reasoning when writing out your solutions.

# Section 2.6 Limits Involving Infinity; Asymptotes of Graphs

## Topics

We will cover these topics in this section.

1. Limits at infinity.
2. Horizontal, oblique, and vertical asymptotes.

## Learning Objectives

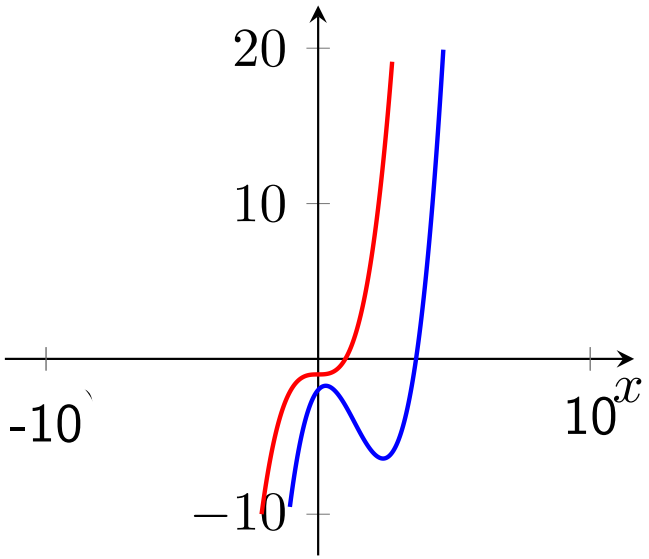
For the topics in this section, students are expected to be able to:

1. Evaluate infinite limits.
2. Identify and sketch horizontal, oblique, and vertical asymptotes of functions.

Students are not expected to apply the formal definition of limit.

# Building Intuition

Suppose $f(x) = x^3 - 4x^2 + 2x - 2$ and $g(x) = x^3 - 1$.




What do you think

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

represents? What is it equal to?

*Hint: plot these functions in Desmos or some other graphing software.*

# Example 1

Compute the limit.

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{x^3 - 4x^2 + 2x - 2}{x^3 - 1}$$

# Definitions

**Definition: Limits at Infinity**

$f(x)$ has **limit** $L$ **as** $x$ **approaches infinity** when

$$\lim_{x \to \infty} f(x) =$$

$f(x)$ has **limit** $L$ **as** $x$ **approaches negative infinity** when

$$\lim_{x \to -\infty} f(x) =$$

**Note:** all the limit laws (from section 2.2) apply to limits at infinity.

# Horizontal Asymptotes

**Definition: Horizontal Asymptote**

The line $y = b$ is a **horizontal asymptote** of $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = b, \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b$$

**Example 2**: Identify the horizontal asymptotes of

$$\frac{7x^3 + 2}{|x|^3 + x + 1}$$

# Oblique Asymptotes

**Definition: Oblique Asymptote**

$f(x)$ has an **oblique asymptote** if it approaches a line of the form

$$y = mx + b$$

for $m \neq 0$, as $x \to \infty$, or as $x \to -\infty$.

**Note**: if $f$ is a rational function, and the degree of its numerator polynomial is 1 greater than the degree of its denominator polynomial, then $f$ has an **oblique asymptote**.

# Example

Identify the oblique asymptotes of

$$\frac{x^2+3}{x-2}$$

# Vertical Asymptotes

**Definition: Vertical Asymptote**

The line $x = a$ is a **vertical asymptote** of $y = f(x)$ if either

$$\lim_{x \to a^+} f(x) = \pm\infty, \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty$$

**Example 3**: Identify the asymptotes of $f(x)$.

$$f(x) = \frac{2}{x^2 - 2}$$

Sketch the graph of $f(x)$.