Section 2.5 : Continuity

Chapter 2 : Limits and Continuity

Math 1551, Differential Calculus

Section 2.5 Continuity

Topics

1. Continuity:
   a) at a point
   b) left and right continuity
   c) on an interval
   d) of functions
   e) of polynomial, rational, composite, and inverse functions
2. Intermediate value theorem (IVT)

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Determine whether, and where, a function is continuous.
2. Extend functions to be continuous at a point
3. Apply the intermediate value theorem to characterize functions.

Definition

Definition: Continuity at a Point

$f(x)$ is **continuous at** $x = c$ if $\lim_{x \to c} f(x) =$

Example 1

a) Give an example of a function that is not continuous at $x = 0$.

b) Give an example of a function that is continuous at $x = 0$.

Definitions

Definition: Left and Right Continuity

$f(x)$ is **left-continuous at** $x = c$ if $\lim_{x \to c^-} f(x) =$

$f(x)$ is **right-continuous** at $x = c$ if $\lim_{x \to c^+} f(x) =$

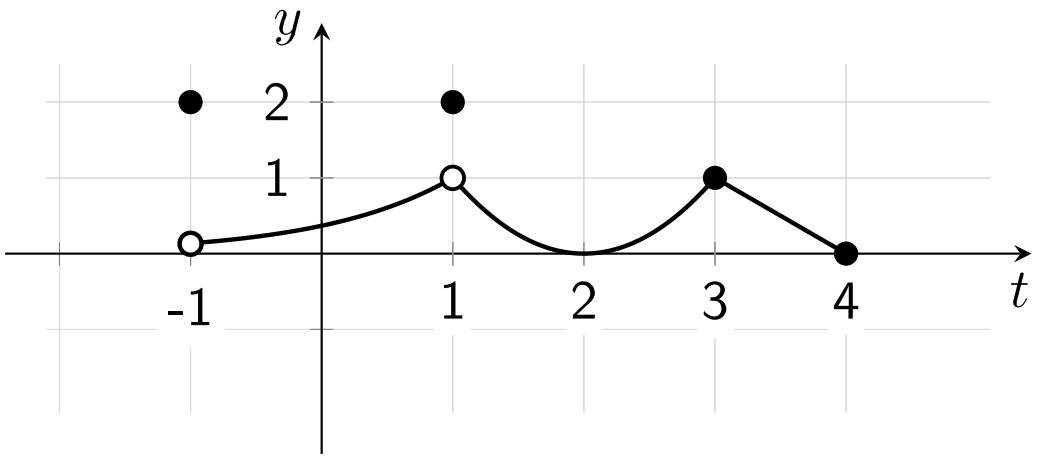
Definition: Continuity on an Interval

$f(x)$ is **continuous on** $[a, b]$ if it is:

- left continuous at
- right continuous at
- continuous at all other points of

Example 2

Where is $y(t)$ continuous on $[-1, 4]$?



Definition

Definition: Continuous Function

A **continuous function** is a function that is continuous everywhere on its domain.

Example 3

Give an example of a function that is continuous.

Polynomial, Rational, Inverse, Composite Functions

It can be shown that **polynomial**, **rational**, **inverse**, and **composite functions** are continuous on their domains.

Example 4

Where are the functions continuous?

a) $f(x) = x^3 + x^2 + x + 1$

b) $g(t) = \dfrac{1}{t-1}$

c) the inverse function of e^x

Example 5

Suppose it takes us about 5 hours to drive from Atlanta to Savannah, a 250 mile road trip. Let $d(t)$ be the distance between us and our destination, as a function of time, t.

a) What are $d(0)$ and $d(5)$ equal to?

b) Is d a continuous function?

c) Is there a time when $d = 100$?

The Intermediate Value Theorem (IVT)

Theorem: IVT

If f is a continuous function on $[a, b]$, $f(a) \leq y_0 \leq f(b)$, then $y_0 = f(c)$ for some $c \in [a, b]$.

Example 6

True or false: if a continuous function is never zero on $[a, b]$, then f does not change sign on that interval.

Continuous Extension (if time permits)

Example 7

Define $g(0)$ in a way so that

$$g(t) = \frac{\sin(t)}{t}, \quad t \neq 0$$

is continuous at the origin.

In-Class Participation Activity: Worksheet

(if time permits)

- Please solve worksheet problems in groups of 2 or 3 students
- Each group submits **one** completed worksheet
- **Clearly print** full names at the top of your sheet
- Every student in a group gets the same grade
- Grading scheme per question:
  - 0 marks for no work or for students working by themselves
  - 1 mark for starting the problem or for a final answer with insufficient justification
  - 2 marks for a complete solution