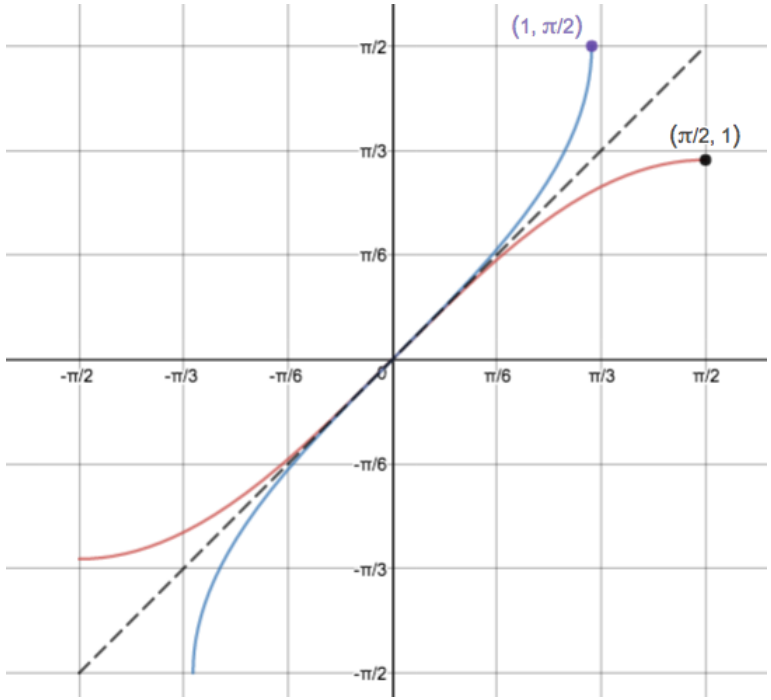
Section 1.6 : Inverse Functions and Logarithms

Chapter 1 : Functions

Math 1551, Differential Calculus



the sine function (red) and the inverse sine function (blue)

1.6 Inverse Functions and Logarithms

Topics

This section reviews material covered in a pre-requisite course. We will review these topics in this section.

1. Inverse functions
2. Logarithmic functions
3. The inverse sine and cosine functions

Learning Objectives

For the topics in this section, students are expected to be able to:

1. Determine whether a function has an inverse, and if it does, find the inverse.
2. Evaluate and simplify expressions involving logarithms and inverse sine and cosine functions.
3. Find the domain and range, and sketch composite functions that incorporate logarithms and inverse sine and cosine functions.

Inverses

Functions f and g are **inverses** if

$$(f \circ g)(x) = (g \circ f)(x) = \_\_\_\_\_$$

Notation $g(x) = f^{-1}(x)$.

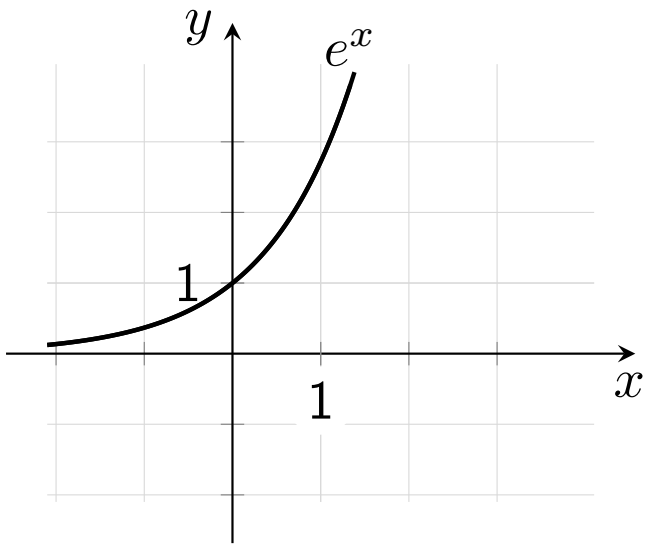
Note that if (x, y) lies on the graph of f, then the point (y, x) lies on the graph of its inverse.

Constructing an Inverse Function

To construct the inverse of $y = f(x)$:

1. _____
2. Solve for y to obtain the inverse function.
3. Replace y with $f^{-1}(x)$.

The graph of an inverse function is obtained by reflecting the curve through the line $y = x$. For example:



Do All Functions Have an Inverse?

An example of a function that does **not** have an inverse is:

We can only invert functions that are _____.

Functions that are _____ will:

Logarithmic Functions

Definition

The **logarithm** with base $a > 0$, $\log_a x$, is the inverse of the base a exponential function, a^x.

The domain of $\log_a x$ is:

The range of $\log_a x$ is:

$\log_e x =$

Properties of Logarithms

$$\log_a(xy) =$$

$$a^{\log_a x} =$$

$$\log_a\left(\frac{x}{y}\right) =$$

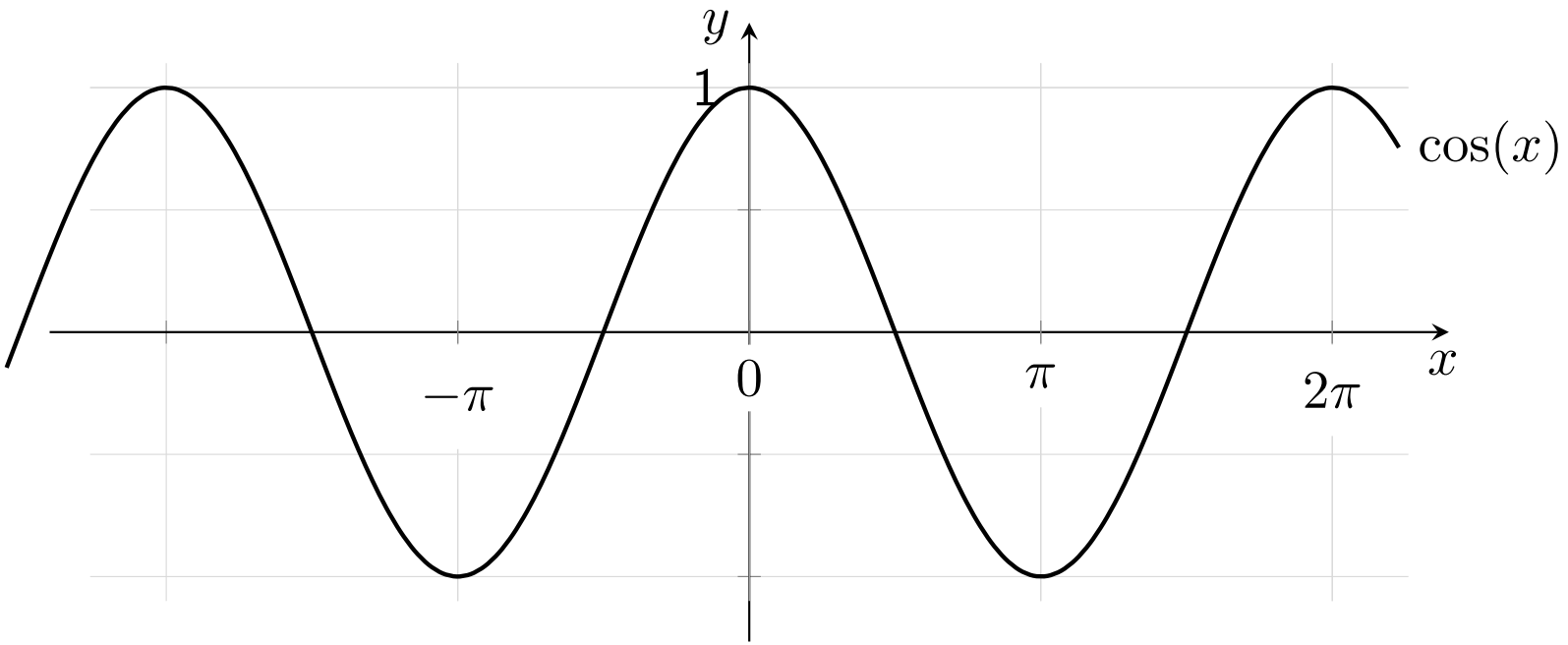
$$\log_b b^x =$$

Example: evaluate the expressions below.

a) $\ln \ln e^e$ b) $e^{-\ln 2}$

Inverse Cosine Function

Is $\cos(x)$ invertible over the domain $(-\infty, \infty)$? Why/why not?

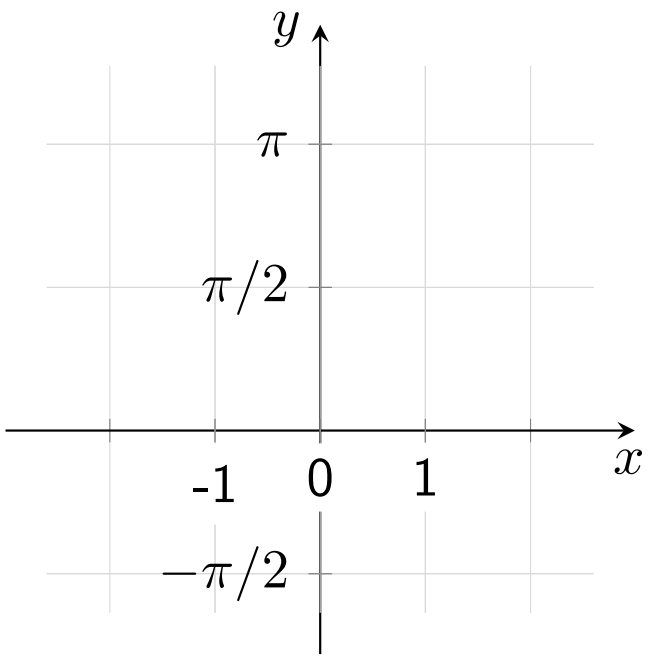


Inverse sine and cosine functions

$\cos^{-1} x$

domain:

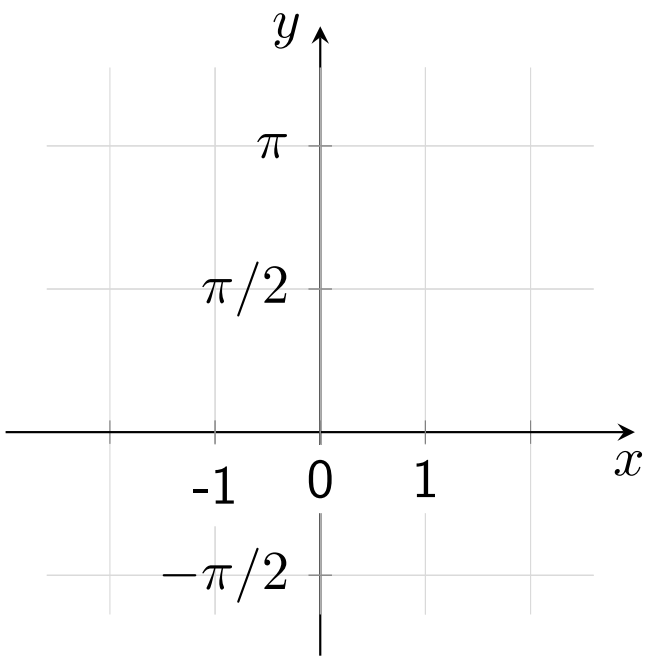
range:



$\sin^{-1} x$

domain:

range:



Examples (as time permits)

More examples will be explored during recitation.

1. Evaluate the following, if possible.
   a) $\cos^{-1} \frac{\pi}{2}$
   b) $\cos^{-1} \frac{1}{\sqrt{2}}$
2. State the domain of the composite functions.
   a) $f(x) = \sin(\sin^{-1} x)$
   b) $g(x) = \sin^{-1}(\sin x)$