Section 1.1 : Functions and Their Graphs

Chapter 1 : Functions

Math 1551, Differential Calculus

Section 1.1 Functions and Their Graphs

Topics

We will cover these topics in this section.

1. functions and the vertical line test
2. domain and range, even/odd functions, increasing/decreasing intervals
3. piecewise functions

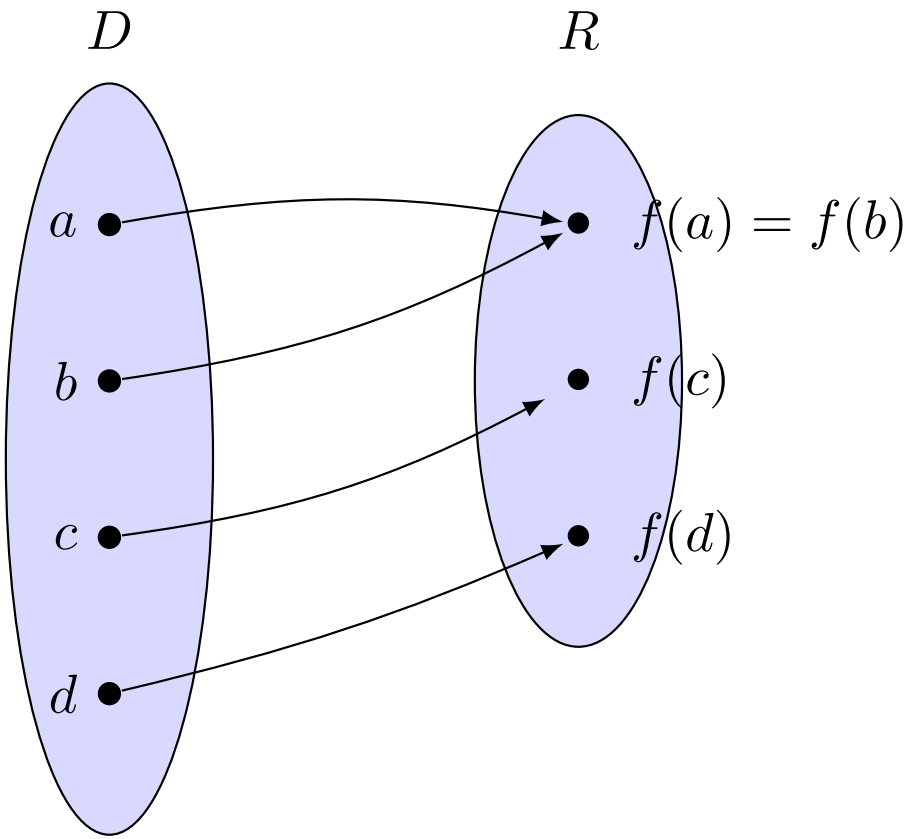
Learning Objectives

For the topics in this section, students are expected to be able to:

1. determine whether a graph is a function
2. characterize functions using domain and range, symmetry

Functions

A **function** is a rule that assigns a **unique** (or single) element $f(x) \in R$ to each element $x \in D$.



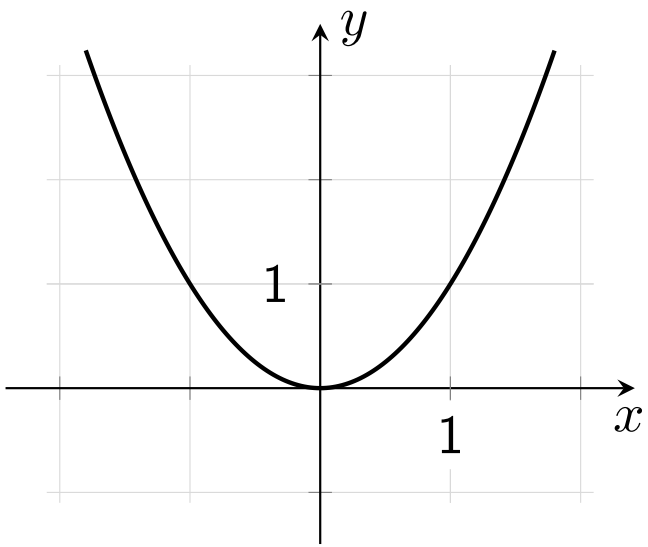
Vertical Line Test

Functions must satisfy the vertical line test.

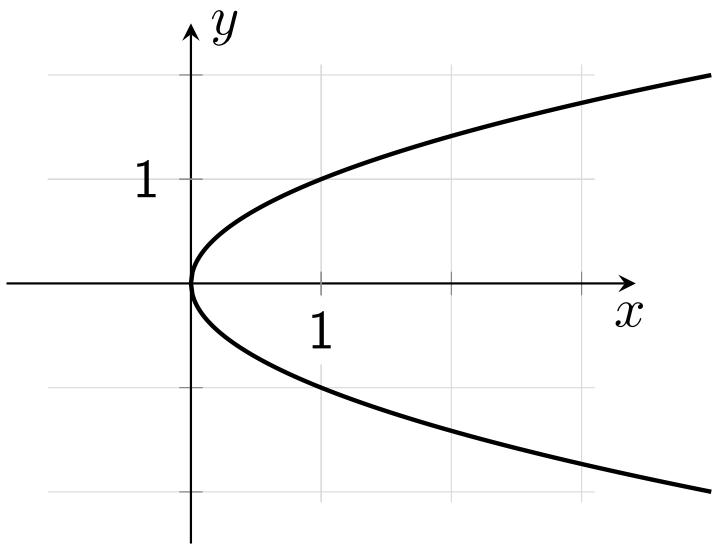
> Any vertical line intersects the graph of a function **at most** once.

Example 1

Which of the following curves represent a function?



$y = x^2$



$y^2 = x$

Domain

Suppose $y = y(x)$. The **domain** of y consists of all numbers that x can have. To find the domain, look for:

1. Zeros in the denominator
2. Negative numbers under a radical (for even roots)
3. Physical restrictions to the problem (e.g. area can't be negative)

Example 2

What is the domain of $y(x)$?

$$y(x) = \sqrt{x - 1}$$

Range

Suppose $y = y(x)$. The **range** is the set of all values that y can have. To find the range, either:

- look at a graph of the function, or
- solve the equation for x, and find any restrictions on y (equivalently, find the domain of the inverse function).

Example 4

What is the range of $y(x)$?

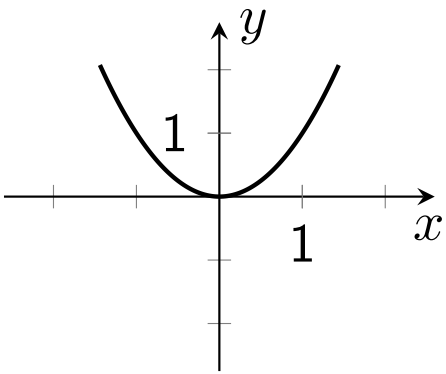
$$y(x) = \sqrt{x - 1}$$

Even and Odd Functions

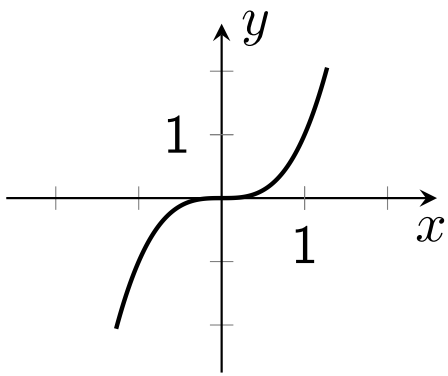
Even functions are symmetric about the y-axis. Odd functions are symmetric about the origin.

Example 5

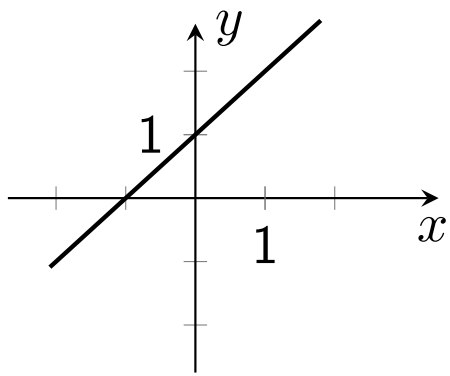
Which of the functions are even? Which are odd?



$y = x^2$



$y = x^3$



$y = x + 1$

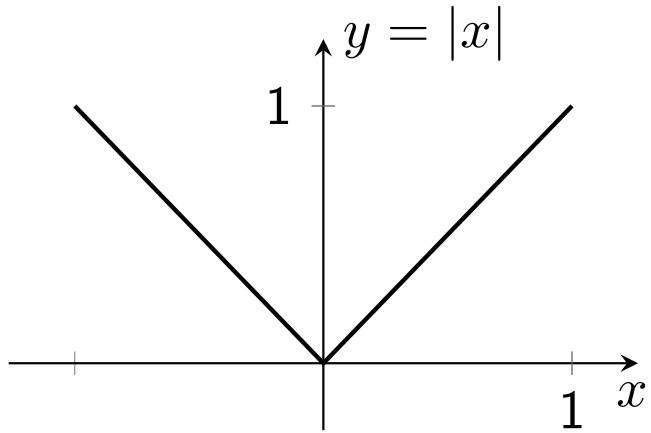
Piecewise Functions

A piecewise function defines a different function over certain intervals.

Example 6

Recall that

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$



Sketch the function $f(x) = 1 + |x - 2|$. What is the range of f?