Math 1551-G **Name:** _____
Fall 2015
Practice Exam 4
20 November 2015
Time Limit: 50 Minutes

This exam contains 7 pages (including this cover page) and 5 questions. There are 48 points in total. Write explanations clearly and in complete thoughts. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	9	
2	9	
3	12	
4	9	
5	9	
Total:	48	

Formal Symbols Crib Sheet

$f : A \to B$	function with domain A & codomain B	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
f^{-1}	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x\to a}$	limit as x approaches a	$\mathbb{R}$	real numbers
$\lim_{x\to a^-}$	limit from below	(a, b)	open interval a to b
$\lim_{x\to a^+}$	limit from above	$[a, b]$	closed interval a to b
$\subset$	subset of	$\in$	element of
$\cap$	intersection	$\cup$	union
$\mapsto$	maps to	f'	derivative
$\frac{d}{dx}$	derivative with respect to x		

Derivatives Crib Sheet

For constant $a \in \mathbb{R}$ and arbitrary real functions f and g

Function	Derivative	Function	Derivative
a	0	af	af'
$f+g$	$f'+g'$	fg	$f'g+fg'$
$\frac{f}{g}$	$\frac{f'g-fg'}{g^2}$	$f\circ g$	$(f'\circ g)g'$
f^{-1}	$\frac{1}{f'\circ f^{-1}}$	x^a	ax^{a-1}
a^x	$a^x \ln a$	$\log_a \lvert x\rvert$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$	$\csc x$	$-\csc x \cot x$
$\cos x$	$-\sin x$	$\sec x$	$\sec x \tan x$
$\tan x$	$\sec^2 x$	$\cot x$	$-\csc^2 x$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$\operatorname{arccsc} x$	$\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	$\operatorname{arcsec} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\arctan x$	$\frac{1}{1+x^2}$	$\operatorname{arccot} x$	$\frac{-1}{1+x^2}$
$\sinh x$	$\cosh x$	$\cosh x$	$\sinh x$

Geometry Crib Sheet

Pythagorean Identity $a^2+b^2=c^2$

Circle: radius r	$A=\pi r^2$	$c=2\pi r$
Box: dimensions x, y, z	$V=xyz$	$A=2(yz+xz+xy)$
Sphere: radius r	$V=\frac{4}{3}\pi r^3$	$A=4\pi r^2$
Right pyramid: height h dim x, y	$V=\frac{1}{3}hxy$	$A=xy+x\sqrt{(y/2)^2+h^2}+y\sqrt{(x/2)^2+h^2}$
Cylinder: height h radius r	$V=\pi hr^2$	$A=2\pi r(h+r)$
Right Cone: height h radius r	$V=\frac{\pi}{3}hr^2$	$A=\pi r\left(r+\sqrt{r^2+h^2}\right)$
Torus: radii $R>r$	$V=2\pi^2r^2R$	$A=4\pi^2rR$
Tetrahedron: edge x	$V=\frac{1}{6\sqrt{2}}x^3$	$A=\sqrt{3}x^2$
Octahedron: edge x	$V=\frac{\sqrt{2}}{3}x^3$	$A=2\sqrt{3}x^2$
Dodecahedron: edge x	$V=\frac{15+7\sqrt{5}}{4}x^3$	$A=3\sqrt{20+10\sqrt{5}}x^2$
Icosahedron: edge x	$V=\frac{5\left(3+\sqrt{5}\right)}{12}x^3$	$A=5\sqrt{3}x^2$

1. (a) (3 points) State the Extreme Value Theorem.

(b) (6 points)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	9	-4	8	3
1	5	1	7	7

Suppose that f is real function with all derivatives existing. The table above shows the values of the first three derivatives at 0 and 1. Which of the following must be true? Circle all that apply.

A. f achieves the value 6 in the interval $(0, 1)$
B. f has a critical point in $(0, 1)$
C. f is concave up on all of $(0, 1)$
D. f is concave down on all of $(0, 1)$
E. f is concave up on some of $(0, 1)$
F. f is concave down on some of $(0, 1)$
G. f has a local minimum in $(0, 1)$
H. f''' achieves the value -1 in $(0, 1)$

2. (9 points) Let $g : \mathbb{R} \to \mathbb{R}$ be the differentiable real fucntion defined by

$$g(x) = \frac{\sqrt{3}}{2} - 2\sin\left(\frac{x}{2}\right)$$

Find the critical points, where g is increasing, and where g is decreasing.

The critical point(s) of g is (are) _____

g is increasing on _____

g is decreasing on _____

3. (12 points) Let $f(x) = \sqrt{3}\cos(5x) + \sin(5x)$ be defined on the domain $[0, 2\pi/5]$. Compute the maximum, minimum and the x-values at which they occur.

   The maximum is _____ which occurs at $x =$_____.

   The minimum is _____ which occurs at $x =$_____.

4. (9 points) Find all the critical points of the function $h(x) = |x^2 - 5x + 6|$. Decide if each point is a local minimum, local maximum, or neither.

5. (9 points) Let $g : \mathbb{R} \to \mathbb{R}$ be the differentiable real fucntion defined by

$$g(x) = \sin x + 10x + 1000$$

Find the inflection points, where g is concave up, and where g is concave down.

The inflection point(s) of g is (are) _____

g is concave up on _____

g is concave down on _____