Math 1551-G **Name:** _____
Fall 2015
Exam 3
30 October 2015
Time Limit: 50 Minutes

This exam contains 7 pages (including this cover page) and 6 questions. There are 40 points in total. Write explanations clearly and in complete thoughts. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	6	
2	8	
3	8	
4	8	
5	5	
6	5	
Total:	40	

Formal Symbols Crib Sheet

$f : A \to B$	function with domain A & codomain B	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
f^{-1}	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x\to a}$	limit as x approaches a	$\mathbb{R}$	real numbers
$\lim_{x\to a^-}$	limit from below	(a, b)	open interval a to b
$\lim_{x\to a^+}$	limit from above	$[a, b]$	closed interval a to b
$\subset$	subset of	$\in$	element of
$\cap$	intersection	$\cup$	union
$\mapsto$	maps to	f'	derivative
$\frac{d}{dx}$	derivative with respect to x		

1. (a) (3 points) Compute the differential $dx^3 e^{x^3}$.

(b) (3 points) What is the first order approximation about a point $a \in \mathbb{R}$ to a differentiable function f?

2. (a) (3 points) Let $f(x) = x^3 + e^{-x^2}$. Find the derivative of the inverse function $f^{-1}(x)$ at the point $x = 125 + e^{-25}$.

   (b) (5 points) Recall that the hyperbolic sine and cosine are the functions $\sinh t = \frac{e^t - e^{-t}}{2}$ and $\cosh t = \frac{e^t + e^{-t}}{2}$. The inverse function to sinh is the 'area hyperbolic sine' denoted arsinh. Give a formula for the derivative of arsinh in terms of sinh, cosh, and arsinh.

3. (a) (3 points) Find an expression for $\frac{dy}{dx}$ in terms of y and x if

$$e^{x \sin y} = y$$

(b) (5 points) Find an expression for the second derivative $\frac{ds^2}{dt^2}$ in terms of s and t if

$$\arctan\left(\frac{s}{t}\right) = t^2$$

4. (a) (5 points) Use a first order approximation to find a rational approximation for $99^{\frac{1}{4}}$.

(b) (3 points) Estimate the percent error in your approximation.

5. (5 points) A conical tank is 5 meters tall with a radius of 3 meter. Water flows into the tank at a rate of 1 L per minute. How fast is the water level rising when the water level is at 1 meter?

6. (5 points) Suppose a bubble consists of a constant volume of 2 mL of fluid forming a spherical shell around a concentric inner sphere filled with air. I inflate my bubble with an air pump that pumps at a constant rate of 5 cc/sec. I find that bubbles pop between 12 and 14 seconds of inflating. Estimate the thickness and radius of a bubble when it pops and the uncertainty in these quantities.