Name: \_\_\_\_\_

This exam contains 7 pages (including this cover page) and 5 questions. There are 31 points in total. Write explanations clearly and in complete thoughts. No calculators or notes may be used. Put your name on every page.

Gra	de Table	
Question	Points	Score
1	4	
2	6	
3	8	
4	9	
5	4	
Total:	31	

Formal Symbols Crib Sheet

$f: A \to B$	function with domain $A$ & codomain $B$	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
$f^{-1}$	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x \to a}$	limit as $x$ approaches $a$	$\mathbb{R}$	real numbers
$\lim_{x \to a^-}$	limit from below	(a,b)	open interval $a$ to $b$
$\lim_{x \to a^+}$	limit from above	[a,b]	closed interval $a$ to $b$
$\subset$	subset of	$\in$	element of
$\cap$	intersection	U	union
$\mapsto$	maps to		

- 1. Consider the function  $f(x) = 5\sin(x/8) + 4$ 
  - (a) (2 points) What is the average rate of change of f on the interval [-2, 4]?

(b) (2 points) Compute the instantaneous rate of change of f at the point x = 0.

2. (a) (3 points) Find the limit

$$\lim_{x \to -\infty} \sqrt{\log_2 |1 - x^3| - \log_2 |x^2 + 3x^3 - 42|}$$

(b) (3 points) Sketch the graph of the function  $2 \arcsin(3x+1) + 1$ .

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- 3. Consider the functions  $g(t) = \arccos(t^2)$  and  $h(t) = \frac{1}{\sqrt{t+4}}$ 
  - (a) (2 points) What is the domain of g?

Domain of g =

(b) (2 points) What is the range of g?

Range of g =

(c) (2 points) Is g invertible? Why or why not?

(d) (2 points) Give the composition function  $g \circ h$  in terms of t.

 $g \circ h(t) =$ 

- Name:
- 4. Consider the piecewise defined real function g

$$g(x) = \begin{cases} \sec\left(\frac{1}{x}\right) & \text{if } x < 0\\ \tan(\pi x) & \text{if } 0 \le x \le 5\\ \frac{x-4}{\sqrt{x+2}} & \text{if } x > 5 \end{cases}$$

(a) (1 point) Give a real number j where g has a jump singularity.

$$j =$$

- (b) (1 point) Give a real number a where g has an oscillating singularity.
  - a =
- (c) (1 point) Give a real number p where g has a pole/divergent singularity.
  - p =

$$g(x) = \begin{cases} \sec\left(\frac{1}{x}\right) & \text{if } x < 0\\ \tan(\pi x) & \text{if } 0 \le x \le 5\\ \frac{x - 100}{\sqrt{x - 10}} & \text{if } x > 5 \end{cases}$$

(d) (2 points) Give a real number r where g has a removable discontinuity. What is the value of  $\lim_{x\to r} g(x)$ ?

r =

$$\lim_{x \to r} g(x) =$$

(e) (2 points) Compute the limit

 $\lim_{x \to +\infty} g(x) =$ 

(f) (2 points) Compute the limit

 $\lim_{x \to 5^-} g(x) =$ 

5. (a) (2 points) State the Squeeze Theorem of Limits.

(b) (2 points) Suppose that f is a function such that

$$-\frac{1}{2}x^2 + 4x - 5 \le f(x) \le 3\frac{\sin(x-4)}{x-4}$$

for all  $x \in (3,5)$ . Explain what you can conclude about  $\lim_{x\to 4} f(x)$ .