Math 1551-G **Name:** _____
Fall 2015
Exam 2
18 September 2015
Time Limit: 50 Minutes

This exam contains 6 pages (including this cover page) and 5 questions. There are 45 points in total. Write explanations clearly and in complete thoughts. No calculators or notes may be used. Put your name on every page.

Grade Table

Question	Points	Score
1	6	
2	9	
3	12	
4	13	
5	5	
Total:	45	

Formal Symbols Crib Sheet

$f : A \to B$	function with domain A & codomain B	$\mathbb{N}$	natural numbers
$f \circ g$	composition of functions	$\mathbb{Z}$	integers
f^{-1}	inverse function	$\mathbb{Q}$	rational numbers
$\lim_{x \to a}$	limit as x approaches a	$\mathbb{R}$	real numbers
$\lim_{x \to a^-}$	limit from below	(a, b)	open interval a to b
$\lim_{x \to a^+}$	limit from above	$[a, b]$	closed interval a to b
$\subset$	subset of	$\in$	element of
$\cap$	intersection	$\cup$	union
$\mapsto$	maps to	$\frac{d}{dx}$	derivative

1. Consider the function $f(x) = \sqrt{x+1}$ and $g(x) = 3^x$

   (a) (3 points) What is the derivative of $f \circ g$ at 0?

   $(f \circ g)'(0) =$

   (b) (3 points) What is the derivative of the inverse of f at 4?

   $(f^{-1})'(4) =$

2. (a) (3 points) Compute the derivative

$$\frac{d}{dx}\left(5^{x\cos^2 x}\right) =$$

(b) (3 points) Compute the derivative

$$\frac{d}{dx}\ln\left|\frac{x^2+1}{x^3-2}\right| =$$

(c) (3 points) Compute the derivative

$$\frac{d}{dx}x^3\tan(x^3) =$$

3. Consider $h(x) = \arctan(5x + 2)$.

   (a) (6 points) Find the third derivative $h'''(x)$.

   (b) (6 points) h is invertible for $x > \frac{1}{\sqrt{15}}$. Compute the value of the inverse derivative $(h^{-1})'$ evaluated at $h(1)$.

4. Consider the piecewise defined real function g

$$g(x) = \begin{cases} x^2 + 4x + 4 & \text{if } x \leq -2 \\ 4\cos\left(\frac{x+2}{2}\right) - 4 & \text{if } -2 < x < \pi/2 - 2 \\ -\sqrt{2x-5} & \text{if } x > \pi/2 - 2 \end{cases}$$

(a) (5 points) Find the domain of the derivative g'

(b) (8 points) Consider the hyperbola defined by the points (x, y) in the plane such that

$$\frac{x^2}{7} - \frac{y^2}{3} = 1$$

Find the equation of the line tangent to the hyperbola at the point $\left(\frac{-7}{\sqrt{3}}, 2\right)$.

5. (5 points) Compute the limit. (Hint: This limit is a derivative of some function at a point. Compute the derivative and use it to evaluate the limit.)

$$\lim_{x \to 0} \frac{\cos \sqrt{25 - x} - \cos 5}{x}$$